NUCLEAR TRAINING COURSE

COURSE 1A25

Level 1A
2 - Science Fundamentals
5 - Heat and Thermodynamics

Index

1A25.00-1 Fundamental Concepts
1A25.00-2 First Law of Thermodynamics
1A25.00-3 Solids, Liquids and Vapours
1A25.00-4 Perfect Gas
1A25.00-5 Variable Specific Heats of Gases
1A25.00-6 Pressure, Volume, Temperature Relationships of Gases
1A25.00-7 The Second Law of Thermodynamics and Entropy
1A25.00-8 Mixtures of Ideal Gases and Vapours
1A25.00-9 Gas Compressors and Engines
1A25.00-10 Vapour Power Cycles, Steam Engines and Steam Turbines

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Course 1A25
Lesson 1A25.00-1

Heat & Thermodynamics

FUNDAMENTAL CONCEPTS

Competence in the field of engineering thermodynamics, necessary for the correct analysis of processes involving energies and the conversion of one form of energy into another, is based upon a thorough knowledge of several fundamental concepts. These fundamentals are briefly discussed in the following paragraphs.

Thermodynamics is the service that deals with energy and its transformation.

Energy may be broadly defined as the ability to produce, or capacity for producing an effect, and may be classified for the purposes of engineering thermodynamics as follows.

Stored Energy

(a) Potential Energy Energy possessed by matter because of its vertical displacement relative to an arbitrary datum plane. Potential energy = weight x height.

(b) Kinetic Energy Energy possessed by matter due to its velocity. Kinetic energy =

$$\frac{\text{weight x (velocity)}^2}{2g} \text{ or } \frac{\text{mass x (velocity)}^2}{2}$$

(c) Internal Energy U Energy possessed by matter due to its activity and configuration of its molecules.

(d) Flow work or flow energy Energy possessed by a fluid flowing in a conduit because of the work done upon it by the fluid upstream causing it to flow across an arbitrary plane. Flow energy per pound of fluid may be evaluated by the product of its pressure and specific volume; flow energy = pv.

(e) Chemical Energy The energy possessed by matter because of its atomic structure, and evidenced by the energy resulting from a chemical reaction.

(f) Nuclear Energy The energy binding the particles within atomic nuclei and evidenced by the energy released during the redistribution of the particles within the atomic nuclei. The relationship between the decrease in mass during a nuclear reaction and the energy released as given by Einstein's equivalence relationship is:

$$E = mc^2$$

where E = energy generated in ergs, m = mass decrease in grams and c = velocity of light, 2.9979×10^{10} cm/sec.

Energy in Transition

(a) Heat, Q Energy in transition from one body to another due to a temperature difference. The quantity of heat is usually evaluated by the product of the weight of the substance, the heat capacity, and the temperature change, or

$$Q = w\int cdT$$

(b) Work, W Translatory energy resulting from the action of a force moving through a distance. It is evaluated by the product of the force and the collinear distance, through which it acts, or $W = \int Fdx$. In the case of a medium undergoing a reversible expansion or contraction, the work is evaluated by the equation:

$W = \int (pdV)_R$, where the subscript R signifies a reversible process.

The thermodynamic system is the region enclosing the matter involved in the energy transformation being studied. The system is separated from the surroundings, the region outside of the system, by an imaginary or real line or envelope termed the boundary. The system will be termed closed when no mass crosses the boundary, or open when there is mass exchange between the system and surroundings. The boundary may be fixed or elastic. The solution of many thermodynamic problems dictates the limitation of the system to the working fluid or medium, while other problems are easily solved by including various pieces of equipment as well as the working fluid in the system.

Equilibrium A system is said to be in equilibrium when there is no tendency of the system to undergo a spontaneous change. A system may be in mechanical equilibrium, thermal equilibrium, chemical equilibrium, etc.

The medium is the material substance which absorbs, rejects, or transports energy during a thermodynamic process or cycle. The term working fluid may be used instead of medium. Examples of media or working fluids are the refrigerant in a refrigerator plant, or water and its vapour in a steam power station.

The <u>physical properties</u> of a medium such as its pressure, temperature, specific volume, etc., define its solid, liquid, or gaseous state, and are sometimes termed the defining co-ordinates of the medium.

<u>Mass, m</u> The quantity of matter expressed in mass units. One pound mass is defined as 0.45359 kilogram. One slug mass is 32.1739 pound mass.

<u>Force, F</u> One pound force is that force which will give a one pound mass an acceleration of 32.1739 ft/sec^2, or one slug mass an acceleration of 1 ft/sec^2.

<u>Weight, w</u> The weight of a given mass is the force of the earth's gravitational field upon that mass, and is generally expressed in pound force.

Newton's law may be expressed as $F = \frac{ma}{g_c}$

in which $g_c = 1 \frac{\text{slug ft}}{\text{lb force sec}^2} = 32.1739 \frac{\text{lb mass ft}}{\text{lb force sec}^2}$

Newton's equation written in terms of the local gravitational acceleration to express the weight of a given mass

becomes $w = \frac{mg}{g_c}$

$$w \text{ (lb force)} = \frac{m(\text{slug})g(\text{ft/sec}^2)}{1 \frac{\text{slug ft}}{\text{lb force sec}^2}} = \frac{m(\text{lb mass})g(\text{ft/sec}^2)}{32.1739 \frac{\text{lb mass ft}}{\text{lb force sec}^2}}$$

The equation shows that the weight expressed in pounds force of a given quantity of matter is numerically equal to its mass expressed in pound mass when the value of g is 32.1739. Since the numerical value of g does not vary greatly over the surface of the earth, in general no distinction is made between pound mass and pound force in engineering notation. Throughout these lessons the word pound (lb) will be used to denote both force and mass.

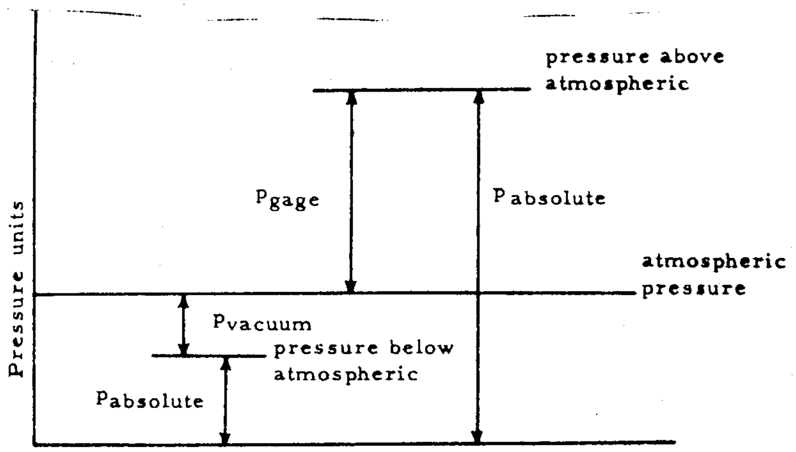
<u>Pressure, p</u>, is defined as force per unit area and may be expressed in pounds per square inch, pounds per square foot, atmospheres, or other units. Fig. 1 illustrates the various common pressure terms.

<u>Density, ρ</u>, is mass per unit volume, and may be expressed as grams per cubic centimeter, pounds per cubic foot, etc.

<u>Specific weight, γ</u>, is the gravitational force exerted on the mass of a medium per unit volume, or weight per unit volume.

Specific volume, v, may be defined as the volume of a unit mass of a medium.

Temperature, T, is an index or a measure of the molecular activity of matter and is expressed in degrees Fahrenheit, Rankine, Centigrade, Kelvin, etc.



Various methods of expressing pressure

Fig. 1

The heat capacity of a medium is the quantity of heat absorbed by a unit quantity of the medium during a temperature change of 1 degree. The ratio of the heat capacity of the medium to the heat capacity of water at a standard state is termed the specific heat of the medium. Since the heat capacity of water is approximately unity, the specific heat of the medium is equal to its heat capacity, and for this reason no distinction is made between heat capacity and specific heat in engineering usage. The quantity of heat required to cause 1 degree change in the temperature of a medium depends upon the process; therefore, the specific heat notation must specify the type of process such as constant-pressure or constant-volume specific heat.

The enthalpy of a medium is defined as the sum of its internal energy per unit weight and the product of its pressure and specific volume, or $h = u + pv/J$.

A process takes place when a system changes from one state to another. Changes in the values of the physical properties are independent of the manner in which the changes occur, and are evaluated by the end points only. This independence has resulted in the use of the term point functions, which is the mathematical way of expressing this condition. The amounts of heat and work depend upon the process; therefore, heat and work are termed path functions, indicating that their values are dependent upon the path or process. The symbols δW and δQ will be used to designate small increments of work and heat, respectively, instead of the usual differential notation.

A process is said to be reversible if, at the conclusion of the process, the system and surroundings can be restored to their initial states and no evidence remains that the process has taken place. The reversible process is sometimes called an ideal process. When a system changes from its initial state to a final state by an irreversible process, evidence of the process remains if the system is restored to its initial state by any reversible process. All natural processes are irreversible.

Thermal efficiency is defined as the ratio of the useful output of a system to the energy input. In the case of prime movers, the thermal efficiency is the ratio of the net work output to the heat supplied.

Zero Law The statement of the zero law of thermodynamics is: Two bodies each on thermal equilibrium with a third body are in equilibrium with each other, and it follows that all three bodies are at the same temperature. This fundamental concept is the foundation of temperature measurement.

First Law When a system undergoes a cyclic process, the algebraic sum of the heat quantities and work quantities are proportional for any given cycle. Stated mathematically the first law is:

$$J\ (\Sigma\,\delta Q)\ \text{cycle} = (\Sigma\,\delta W)\ \text{cycle or } J\oint \delta Q = \oint \delta W$$

Devices which violate this principle are called perpetual motion machines of the first kind.

Second Law The second law may be stated in several ways, two of which are given here. It is impossible to construct a self-acting machine which, operating in a cycle, will produce no effect other than the absorption of heat from one reservoir and the performance of an equivalent amount of work. This is the Kelvin-Planck statement. Another statement is that it is impossible to constuct a device which, operating in a cycle, will product no effect other than the transfer of heat from a body at low temperatures to a reservoir at a higher temperature. Devices which violate this law are called perpetual motion machines of the second kind.

<u>Entropy</u> One consequence of the second law is the introduction of a physical property of matter called entropy, designated by S. The change in entropy of a substance or system during a reversible process may be evaluated by the following equation:

$$ds = \left(\frac{\delta Q}{T}\right)_R \qquad \text{or} \qquad \Delta S = \int_R \frac{\delta Q}{T} \qquad \text{or}$$

$$S_2 - S_1 = \int_R \frac{\delta Q}{T}$$

where the subscript R indicates a reversible process. It must be understood that δQ cannot be evaluated until the process is specified. A more detailed treatment of entropy is presented in a following lesson.

The following examples illustrate the use of fundamental concepts, and should be studied carefully.

<u>Example 1</u>:

A 100-lb weight is located 300 ft above a reference plane. What type of energy does the weight possess and what is its magnitude?

<u>Solution</u>:

The weight possesses potential energy.

Potential energy = 100 lb x 300 ft = 30,000 ft-lb

<u>Example 2</u>:

A truck weighing 10 tons is travelling at sea level with a velocity of 50 mph. What form of energy does the truck possess and what is its magnitude?

<u>Solution</u>:

The truck possesses kinetic energy.

Weight = 10 ton x 2000 lb/ton = 20,000 lb

$$\text{Velocity} = 50\ \frac{\text{mile}}{\text{hr}} \times 5280\ \text{ft/mile} \times \frac{1}{3600\ \text{sec/hr}} = 73.3\ \text{fps}$$

$$\text{Kinetic energy} = \frac{20{,}000\ \text{lb}}{32.2\ \text{ft/sec}^2} \times \left(73.3\ \frac{\text{ft}}{\text{sec}}\right)^2 \times \frac{1}{2}$$

$$= 1{,}670{,}000\ \text{ft-lb}$$

Example 3:

An airplane weighing 4,000 lb, flying at 10,000 ft altitude has a velocity of 250 mph. Based upon the sea level, what is the total energy of the plane?

Solution:

$$\text{Velocity} = 250 \text{ mph} = 367 \text{ fps}$$

$$\text{Weight} = 4{,}000 \text{ lb}$$

$$\text{Kinetic energy} = \frac{4{,}000 \text{ lb}}{32.2 \text{ ft/sec}^2} \times \left(367 \frac{\text{ft}}{\text{sec}}\right)^2 \times \frac{1}{2}$$

$$= 8{,}370{,}000 \text{ ft-lb}$$

$$\text{Potential energy} = 4{,}000 \text{ lb} \times 10{,}000 \text{ ft}$$

$$= 40{,}000{,}000 \text{ ft-lb}$$

$$\text{Total energy} = \text{kinetic energy} + \text{potential energy}$$

$$= 48{,}370{,}000 \text{ ft-lb}$$

Example 4:

Fifty lb water are heated from 60 F to 160 F by addition of 5,000 Btu of energy. What is the final form of the energy?

Solution:

Internal energy of the water.

Example 5:

Air at 200 psia pressure and with a specific volume of 1.4 ft^3/lb is flowing in a conduit. What is the value of the flow energy per pound of air?

Solution:

$$\text{Flow energy} = pv = 200 \frac{\text{lb}}{\text{in}^2} \times 144 \text{ in}^2/\text{ft}^2 \times 1.4 \text{ ft}^3/\text{lb}$$

$$= 40{,}300 \text{ ft-lb/lb}$$

Example 6:

Water at 200 F and a pressure of 800 psia is flowing through a pipe with a velocity of 100 fps. The pipe is at an elevation of 250 ft above a reference plane. If the internal

Example 6:

energy of the water is 170 Btu/lb and its specific volume 0.0166 ft^3/lb, what is the total energy possessed by 1 lb of water?

Solution:

Potential energy = 1 lb x 250 ft = 250 ft-lb/lb

Kinetic energy

$$= (1\ \text{lb}) \left(100\ \frac{\text{ft}}{\text{sec}}\right)^2 \times \frac{1}{32.2\ \text{ft/sec}^2} \times \frac{1}{2} = 155\ \text{ft-lb/lb}$$

Internal energy

$$= 170\ \frac{\text{Btu}}{\text{lb}} \times 778\ \text{ft-lb/Btu} = 132{,}000\ \text{ft-lb/lb}$$

Flow energy

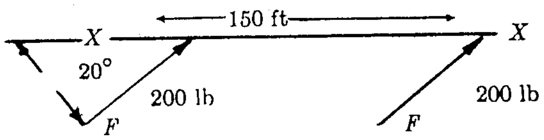
$$= 800\ \frac{\text{lb}}{\text{in}^2} \times 144\ \text{in}^2/\text{ft}^2 \times 0.0166\ \text{ft}^3/\text{lb} = 1{,}920\ \text{ft-lb/lb}$$

134,325 ft-lb/lb

Example 7:

A force of 200 lb acts along a line making an angle of 20° with the X-axis. If the force moves 150 ft along the X-axis, how much work has been done?

Solution:



Force in direction of motion = 200 lb x cos 20° = 188 lb

Work = 188 lb x 150 ft = 28,200 ft-lb

Example 8:

Three hundred gal fuel oil are to be pumped into a storage tank 500 ft above the floor. If the oil has a specific weight of 52 lb/ft^3, (a) how much work must be performed in pumping the oil neglecting losses, and (b) what is the form of the energy at the end of the process?

Solution:

(a) weight per gal = $52 \text{ lb/ft}^3 \times \frac{1}{7.48 \text{ gal/ft}^3} = 6.97 \text{ lb/gal (US)}$

total weight = 6.97 lb/gal x 300 gal = 2,090 lb

force to raise oil = 2,090 lb

work = 2,090 lb x 500 ft = 1,045,000 ft-lb

(b) Final form of energy is potential

Example 9.

Indicate whether these systems are opened or closed.
(a) pump (b) tank of water (c) moderator system

Solution:

(a) open (b) closed (c) closed

Example 10:

A pressure gauge indicates a vacuum of 10 psi when the atmospheric pressure is 28.70 in. Hg. What is the absolute pressure in psi?

Solution:

Barometric pressure

$= 28.70 \text{ in Hg.} \times 0.491 \frac{\text{lb}}{\text{in}^2}\text{/in. Hg} = 14.1 \text{ psi}$

Absolute pressure

$= p_{ab} = 14.1 \text{ psi} - 10 \text{ psi} = 4.1 \text{ psi}$

Example 11:

The pressure of air in a tank as indicated by a gauge is 200 psi when the atmospheric pressure is 14.70 psia. What is the absolute pressure?

Solution:

$p_{ab} = 200 \text{ psi} + 14.70 \text{ psi} = 214.7 \text{ psia}$

Example 12:

Three ft^3 of a material weighs 350 lb. What is the specific weight of the material?

Solution:

$$\gamma = \frac{350 \text{ lb}}{3 \text{ ft}^3} = 117 \text{ lb/ft}^3$$

Example 13:

What is the specific volume of the material of the above problem?

Solution:

$$v = \frac{1}{\gamma} = \frac{1}{117 \text{ lb/ft}^3} = 0.00855 \text{ ft}^3\text{/lb}$$

Example 14:

The temperature of oil measured on the Fahrenheit scale is 70 deg. What is the temperature of the oil in degrees Rankine, Kelvin, and Centigrade?

Solution:

Degree R = degree F + 460 = 70 + 460 = 530 R

Degree K = degree R x 5/9 - 530 x 5/9 = 294 K

Degree C = degree K - 273 = 294 - 273 = 21 C

Check, degree C = (degree F - 32) 5/9 = (70 - 32) 5/9

= 21 C

Example 15:

When 70 Btu of heat are transferred to 10 lb of air at constant pressure, the temperature changes from 50 F to 79.2 F. What is the average specific heat of the air for this temperature range?

Solution:

$$c_p = \frac{Q}{w(t_2 - t_1)} = \frac{70 \text{ Btu}}{10 \text{ lb } (79.2 \text{ F} - 50 \text{ F})} = 0.24 \text{ Btu/lb F}$$

Example 16:

What is the enthalpy of 1 lb steam at 300 psia and 417.3 F when its internal energy is 1117.1 Btu/lb and it has a specific volume of 1.543 ft3/lb?

Solution:

$$h = u + \frac{pv}{J}$$

$$= 1117.1 \text{ Btu/lb} + 300 \frac{\text{lb}}{\text{in}^2} \times 144 \text{ in}^2/\text{ft}^2 \times 1.543 \text{ ft}^3/\text{lb} \times \frac{1}{778 \text{ ft-lb/Btu}}$$

$$h = 1117.1 \text{ Btu/lb} + 86 \text{ Btu/lb} - 1203.1 \text{ Btu/lb}$$

Example 17:

An internal combustion has an output of 10 hp with a fuel consumption of 4.2 lb/hr. If the fuel has a heating value of 20,000 Btu/lb, what is the thermal efficiency of the engine?

Solution:

Output for 1 hour = 10 hp-hr x 2545 Btu/hp-hr

= 25,450 Btu

Heat supplied per hour = 4.2 lb x 20,000 Btu/lb

= 84,000 Btu

$$= \frac{25{,}450 \text{ Btu}}{84{,}000 \text{ Btu}} \times 100 = 30\%$$

ASSIGNMENT

1. A thermodynamic system has a pressure of 110.0 psia and a volume of 2 ft3. During a reversible expansion the volume changes to 5 ft3 and the pressure changes according to the equation p = 140 - 15 V, in which p is pressure in psia, and V is the total volume in ft3. How much work is done by the system?

Hint: $W = \int_{V_1}^{V_2} p\,dV$, in which p is absolute pressure in psf

2. A thermodynamic system expands reversibly at constant pressure of 25 psia from a volume of 20 ft^3 to a volume of 30 ft^3. What work was done by the system?

3. A diesel engine has an efficiency of 37% when burning 18 lb of fuel (18,600 Btu/lb) per hr at full load. What horsepower is the engine developing?

4. When 5 lb copper at 200 F are immersed in 2 lb of water at 60 F, the final temperature of the water and copper is observed to be 86.0 F. What is the specific heat of the copper, considering the specific heat of water to be unity? How much energy was absorbed by the water?

5. Forty lb of a gas per sec. flow through a horizontal 12 in. diameter pipe. The gauge pressure of the gas is 100 psig when the barometric pressure is 29.92 in. Hg. The specific volume of the gas is 0.1 lb/ft^3. Evaluate the forms of energy possible from this data.

6. What forms of energy enter and leave a steam turbine operating in the usual manner?

George Howey

SYMBOLS

The symbols that will be used throughout these lessons are listed below

a	acceleration, linear
A	Area
Btu	British thermal unit
c	specific heat, general
c_v	constant volume specific heat
c_p	constant pressure specific heat
C	Constant
C	degree Centigrade
C_v	constant volume molar heat
C_p	constant pressure molar heat
D	Diameter
f	coefficient of friction
F	Force
F	degree Fahrenheit
g	gravitational constant
h	enthalpy
H	total enthalpy, or molar enthalpy
J	heat equivalent of work
k	specific heat ratio
K	degree Kelvin
m	mass
M	molecular weight or Mach number
n	exponent of polytropic process
n	speed revolutions per unit time
n	number of moles
N	mole fraction
p	pressure
Q	quantity of heat
r	compression ratio
R_o	universal gas constant
R	degree Rankine
R	subscript for a reversible process
s	entropy per unit weight
S	total entropy
t	temperature, degree Fahrenheit
T	absolute temperature
T_o	total temperature
u	internal energy per pound
U	total internal energy or molar internal energy
v	specific volume
V	total volume
V	velocity

SYMBOLS

(cont'd)

w	weight
W	Work
x	quality of vapour
γ	specific weight
Δ	symbol for difference $\Delta p = p_2 - p_1$

η	efficiency
φ	relative humidity
ω	specific humidity
ρ	density
γ	wall force

Course 1A25
Lesson 1A25.00-2

Heat & Thermodynamics

FIRST LAW OF THERMODYNAMICS

It is a matter of experimental observation that for any system undergoing a cyclic process the summation of the work input and output is proportional to the summation of the heat supplied and rejected during one cycle. This concept known as the first law of thermodynamics expressed mathematically is:

$$\Sigma \text{cycle } (\delta W) = J \ \Sigma \text{cycle } (\delta Q) \quad \text{-----------------} (1)$$

or $$\oint \delta W = J \oint \delta Q$$

where J is the constant proportionality factor. The value of J, historically known as Joule's constant, now an international standard is:

$$1 \text{ Btu} = 778.26 \text{ ft-lb}$$

or $$J = 778.26 \text{ ft-lb/Btu}$$

It is common practice to use 778 rather than the more exact value.

When heat and work are expressed in the same system of units by employing appropriate unit conversion factors, equation (1) may be written as:

$$\oint \delta W = \oint \delta Q \quad \text{or} \quad \oint \delta Q - \oint \delta W = 0 \quad \text{------------------} (2)$$

As a consequence of the first law, a property of the system U, known as the internal energy exists. For equation (2) to be valid for all non-cyclic processes, it follows that:

$$dU = \delta Q - \delta W \quad \text{--------------------------------------} (3)$$

for any closed system in the absence of gravitational and magnetic effects. Equations (1), (2) and (3) are applicable to systems undergoing irreversible as well as idealized reversible processes.

Integrating equation (3) for a cyclic process gives:

$$\oint dU = \oint \delta Q - \oint \delta W = 0 \quad \text{--------------------------} (4)$$

Thus it can be seen that the change in the internal energy of a system is zero for any cyclic process. Another way of stating this concept is to say that the change in the internal energy of a system is independent of the process and depends only upon the initial and final states of the system, thus, internal energy is a point function.

The first law of thermodymanics is a powerful tool available for use in the solution of thermodynamic problems. A word statement of the first law applied to any thermodynamic process is:

Energy in + energy stored in the system
at condition 1 = energy out + energy
stored in the system in condition 2 ------(5)

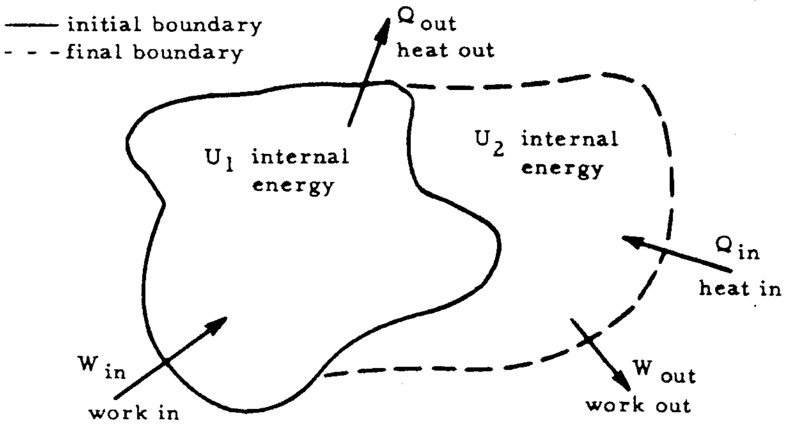
Equation (5) may be used with systems undergoing irreversible as well as the idealized reversible processes.

Before equation (5) can be utilized, the thermodynamic system must be defined. A problem may be simplified by the judicious selection of the system.

The quantities of energy present in various forms, such as heat, potential energy and work, nust be expressed in the same units before they can be summed. By themselves, heat, internal energy and enthalpy are usually expressed in British thermal units (Btu), whereas work, potential energy and kinetic energy are expressed in ft-pounds (ft-lb). The conversion factor (J) is employed to convert one system of units to the other.

Closed System

A large group of the thermodynamic problems can be solved by using the closed system, obtained by defining the system so as to prevent mass transfer across the system's boundaries. This does not preclude mass flow within the system. Some authors prefer to call the energy equation for this system the nonflow energy equation.



Generalized Closed System

Fig. 1

Fig. 1 is a schematic representation of a general closed system. The integrated form of eq. (3) for the closed system is eq. (6).

$$Q - W = U_2 - U_1 \qquad \text{-------------------------}(6)$$

where $Q = \Sigma$ heat terms, and $W = \Sigma$ work terms. Heat is positive if heat enters the system, and negative if the heat leaves the system. Work is positive if work is performed by the system, and negative if work is performed upon the system. When the system is in motion, potential and kinetic energy must be included in the energy equation.

Example 1

A steel tank contains 10 lb air at 120°F having an internal energy of 98.8 Btu/lb. Heat is added to the air until the internal energy of the air increases to 125.6 Btu/lb. What is the amount of heat transferred to the air?

Solution

Choosing the air as the system and applying equation (6) gives:

$$Q - W = 10 \text{ lb } (125.6 \text{ Btu/lb} - 98.8 \text{ Btu/lb})$$

$$W = 0 \text{ (no change in volume)}$$

$$Q = 10 \text{ lb } (26.8) \text{ Btu/lb} = 268 \text{ Btu}$$

Note: If the steel tank and air mass are chosen as the system, additional information concerning the heat transferred to the tank is necessary before the problem can be solved.

Example 2

A system receives 15 Btu of heat and 8500 ft-lb of work. What is the change in internal energy?

Solution

The system receives heat and work therefore Q is + and W is -. Applying equation (6) to the system gives:

$$Q - W = \Delta U$$

$$+ 15 \text{ Btu} - \left(- \frac{8500 \text{ ft-lb}}{778 \text{ ft-lb/Btu}}\right) = \Delta U$$

$$\Delta U = 15 \text{ Btu} + 10.9 \text{ Btu} = 25.9 \text{ Btu}$$

Example 3

The internal energy of a system decreases by 100 Btu while 150 Btu of work are transferred to the surroundings. Is heat added to or taken from the system?

Solution

The internal energy of the system decreases and the system does work; therefore ΔU is -, and W is +. Applying equation (6) to the system gives:

$$Q - (+150\ \text{Btu}) = -100\ \text{Btu}$$

$$Q = (-100 + 150)\ \text{Btu} = +50\ \text{Btu}$$

The answer indicates that the system receives 50 Btu from the surroundings.

Example 4

A quantity of gas under atmospheric pressure is contained in a closed steel tank. An electric resistance heater heats the gas at a uniform rate. Write an equation for the internal energy of the gas at any time T after the current has been turned on.

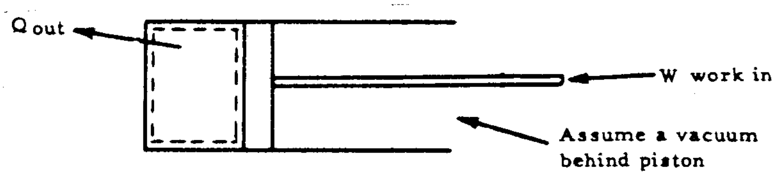
Solution

Choose the gas as the system. Since the volume of the gas is approximately constant, no work is performed on or by the system. Let Q_{in} represent the quantity of heat added, and Q_{out} the quantity of heat lost by the gas. Applying equation (6) to the system gives:

$$Q_{in} - Q_{out} = U_2 - U_1 \quad \text{or} \quad U_2 = Q_{in} - Q_{out} + U_1$$

Example 5

A quantity of gas is compressed behind a movable piston in a cylinder. The piston is released and permitted to move until the volume of the gas is reduced to one-half its original value. It is found that 100 Btu of heat left the gas. If the internal energy of the gas is assumed to be constant, how much work is done on the system which is assumed to be the gas?



Cylinder, Piston, and Compressed Gas

Fig. 2

Solution

Writing equation (6) for the system gives:

$$- Q - W = 0$$

$$- Q = W \quad \text{or} \quad - 100 \text{ Btu} = W$$

$$W = - 100 \times 778 = - 77{,}800 \text{ ft-lb}$$

Example 6

Air at 150 F and 100 psia flows from an insulated pipe through a small valve into an evacuated insulated steel tank. When the pressure in the tank reaches line pressure, the valve is closed. What is the weight of air in the tank if the volume is 5 ft^3?

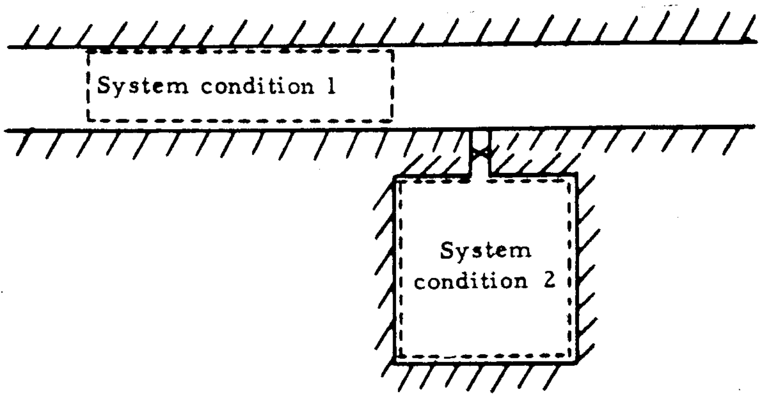
Solution

Choose the air that enters the tank as the system. Write equation (6) for 1 lb air.

$$Q = 0$$

Therefore

$$- W = 1 \text{ lb}(u_2 - u_1)$$



Unsteady Flow Process

Fig. 3

The work done on the 1 lb air entering the tank by the other air in the pipe is equal to -pv since the line pressure is assumed to be constant. Therefore,

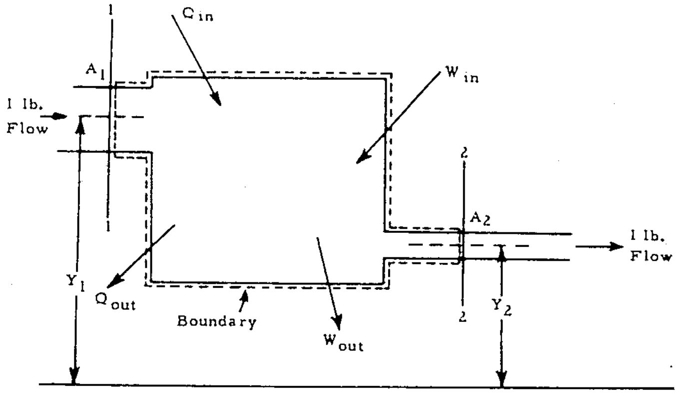
$$- (-p_1 v_1) = u_2 - u_1 \quad \text{or} \quad u_2 = p_1 v_1 + u_1 = h_1$$

From the air table, h_1 at 150 F is 145.88 Btu/lb, but $u_2 = h_1$; Therefore u_2 = 145.88 Btu/lb. From the air tables at u = 145.88, the temperature is 851 R. The specific weight of air at 100 psia pressure and 851 R is 0.316 lb/ft^3; therefore the total weight of air in the tank is equal to 5 ft^3 x 0.316 lb/ft^3 or 1.58 lb.

Open System

Fig. 4 is a sketch of a generalized open thermodynamic system, utilizing one fluid, encircled by a dotted line representing the imaginary boundary. For simplication it will be assumed that the system is stationary and that all electrical, chemical, etc, effects are negligible.

All energy quantities are evaluated at the system's boundaries with the exception of the stored energy. Equation (7) is obtained by applying the first law to an open system with one fluid entering and leaving. w_1 and w_2 represent the weights of fluids entering and leaving, respectively.



Generalized Open System With One Fluid

Fig. 4

$$\overset{\text{Heat}}{Q_{in}} + \overset{\text{Work}}{\frac{W_{in}}{J}} + \overset{\text{Flow energy}}{\frac{w_1 p_1 v_1}{J}} + \overset{\text{Internal energy}}{w_1 u_1} + \overset{\text{Kinetic energy}}{\frac{w_1 V_1^{2}}{J2g}} + \overset{\text{Potential energy}}{\frac{w_1 Y_1}{J}} + \overset{\text{Stored energy}}{E_1} = \overset{\text{Heat}}{Q_{out}}$$

$$+ \overset{\text{Work}}{\frac{W_{out}}{J}} + \overset{\text{Flow energy}}{\frac{w_2 p_2 v_2}{J}} + \overset{\text{Internal energy}}{w_2 u_2} + \overset{\text{Kinetic energy}}{\frac{w_2 V_2^{2}}{J2g}} + \overset{\text{Potential energy}}{\frac{u_2 Y_2}{J}} + \overset{\text{Stored energy}}{E_2} \quad \text{------}(7)$$

It will be noted that all terms are expressed in Btu units. If more than one fluid is involved, additional terms will appear in equation (7).

Equation (7) may also be written for the transient system in terms of differential with respect to time as

$$\frac{\delta Q}{dt} + \left(U_1 + \frac{p_1 v_1}{J} + \frac{\bar{V}_1^{2}}{2gJ} + \frac{Y_1}{J}\right) \frac{dw_1}{dt} =$$

$$\frac{\delta W}{dt} + \left(U_2 + \frac{p_2 v_2}{J} + \frac{\bar{V}_2^{2}}{2gJ} + \frac{Y_2}{J}\right) \frac{dw_2}{dt} + \frac{d(\Delta E)}{dt} \quad \text{---------}(7a)$$

in which $\frac{\delta Q}{dt}$ = net rate of heat transfer

$\frac{\delta W}{dt}$ = net rate of work performed

$\frac{d(\Delta E)}{dt}$ = rate of change of energy storage

$\frac{dw_1}{dt}$ = weight rate of fluid flow in

$\frac{dw_2}{dt}$ = weight rate of fluid flow out

δQ and δW have the sign notation employed in equation (7).

Steady-Flow Energy Equation

The majority of engineering systems are such that the conditions at points 1 and 2, and the values of Q and W do not vary appreciably during a chosen time interval, or the system experiences steady flow conditions. Under steady flow conditions, $w_1 = w_2$, and equation (7) written for one pound of fluid applied to a one-fluid steady flow system becomes:

$$Q_{in} + \frac{W_{in}}{J} + \frac{(p_1 v_1)}{J} + u_1 + \frac{Y_1}{J} + \frac{V_1^2}{2gJ} = Q_{out} + W_{out} + \frac{p_2 v_2}{J} + u_2 + \frac{V_2^2}{2gJ} + \frac{Y_2}{J} \quad \text{------(8)}$$

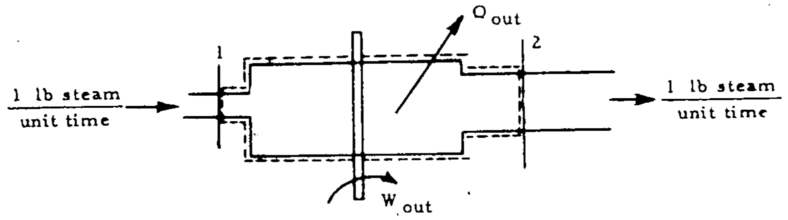
It will be noted that the E terms have disappeared because no energy can be stored within the system under steady flow conditions. Equation (8) may be further simplified by combining the heat terms and work terms, and noting that $\frac{pv}{J} + u = h$.

$$Q - \frac{W}{J} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2gJ} + (Y_2 - Y_1)\frac{1}{J} \quad \text{-------(9)}$$

Q and W will have the same sign notation as in equation (2).

Equation (9) will now be applied to a number of thermodynamic problems.

Example 7



Steam Turbine

Fig. 5

The enthalpies of steam entering and leaving a steam turbine are 1349 Btu/lb and 1100 Btu/lb, respectively. The estimated heat loss is 5 Btu/lb of steam. Neglecting changes in kinetic and potential energy, determine the output of the turbine per pound of steam.

Solution

$$Q - W = (h_2 - h_1)$$

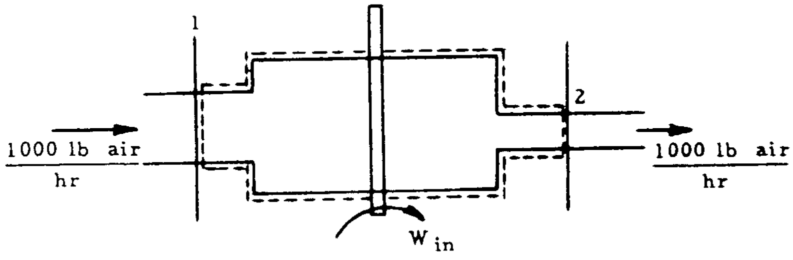
$$-5 - W = 1 \text{ lb } (1100 - 1349) \text{ Btu/lb} = -249 \text{ Btu}$$

$$-W = -249 + 5 = -244 \text{ Btu}$$

$$W = +244 \text{ Btu, or } 244 \text{ Btu} \times 778 \text{ ft-lb/Btu} = 189{,}832 \text{ ft-lb}$$

The positive value indicates that work is being performed by the turbine.

Example 8



Air Compressor

Fig. 6

Air enters a centrifugal compressor with a velocity of 200 fps, a pressure of 1 atm, and an enthalpy of 124.3 Btu/lb. If the air leaves with a velocity of 400 fps, a pressure of 50 psia, and an enthalpy of 185.4 Btu/lb, neglecting heat transfer, determine the work, in horsepower, required to drive the compressor per 1000 lb air per hr.

Solution

Writing equation (9) for 1 lb air gives:

Kinetic energy entering

$$\frac{V_1^2}{2gJ} = \left(200 \frac{ft}{sec}\right)^2 \times \frac{1}{2} \times \frac{1}{32.2 \text{ ft/sec}^2} \times \frac{1}{778 \text{ ft-lb/Btu}} = 0.8 \text{ Btu}$$

Kinetic energy leaving

$$\frac{V_2^2}{2gJ} = \left(400 \frac{ft}{sec}\right)^2 \times \frac{1}{2} \times \frac{1}{32.2 \text{ ft/sec}^2} \times \frac{1}{778 \text{ ft-lb/Btu}} = 3.2 \text{ Btu}$$

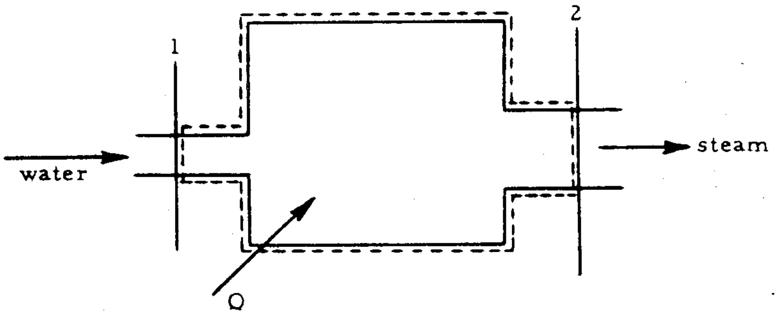
$$-W = (185.4 - 124.3) \text{ Btu} + (3.2 - 0.8) \text{ Btu} = +63.5 \text{ Btu}$$

$$W = -63.5 \text{ Btu/lb air}$$

$$W = -63.5 \text{ Btu/lb air} \times 1000 \text{ lb air/hr} \frac{1}{2545 \text{ Btu/hp-hr}} = -25 \text{ hp}$$

Example 9

Water enters a boiler with an enthalpy of 269.6 Btu/lb, and steam leaves with an enthalpy of 1204.4 Btu/lb. Neglecting changes in velocity and elevation, determine the heat input per hour if the steaming rate is 50,000 lb steam per hr.



Steam Generator

Fig. 7

Solution

Writing equation (9) for the system in terms of 1 lb steam gives:

$$Q = (1204.4 - 269.6) \text{ Btu/lb} = 934.8 \text{ Btu/lb}$$

For 50,000 lb/hr

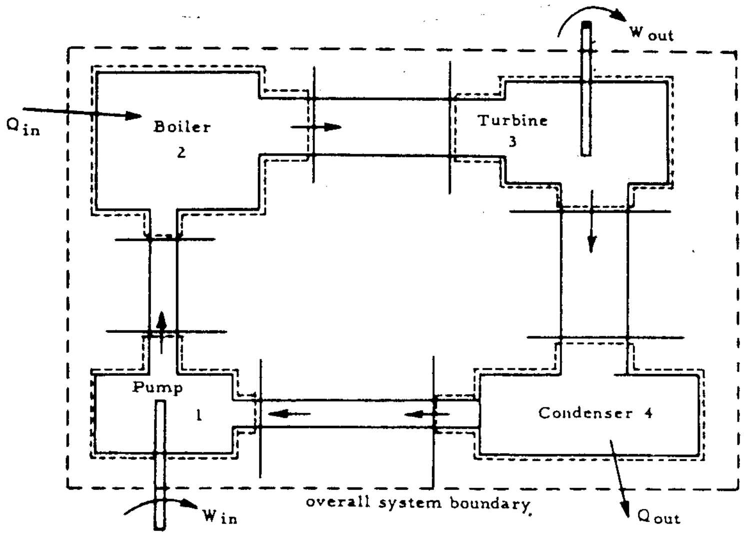
$$Q = 934.8 \text{ Btu/lb} \times 50,000 \text{ lb/hr} = 46,740,000 \text{ Btu/hr}$$

Example 10

A steam power plant consists of a pump, boiler, turbine, and condenser. The enthalpies of the water and steam at various points in the cycle are given below:

		h
Pump	Water in	28.06 Btu/lb
	Water out	29.56 Btu/lb
Boiler	Water in	29.56 Btu/lb
	Steam out	1345.7 Btu/lb
Turbine	Steam in	1345.7 Btu/lb
	Steam out	1020 Btu/lb
Condenser	Steam in	1020 Btu/lb
	Water out	28.06 Btu/lb

Neglecting changes in velocity and elevation, determine the heat added in the boiler, the heat rejected in the condenser, the work input to the pump, the work output of the turbine, the net heat transfer, the net work performed, and the cycle efficiency.



Note when using overall system it must be assumed that there is no matter inside the boundary with the exception of working fluid and the equipment.

Steam Power Plant System

Fig. 8

Solution

Write equation (9) for 1 lb water or steam for each simple system and then for the complete cycle.

Pump system 1

$-W = (29.56 - 28.06)$ Btu/lb $= +1.5$ Btu/lb water;

$W = -1.5$ Btu/lb water

Boiler system 2

$Q = (1345.7 - 29.56)$ Btu/lb $= +1316.1$ Btu/lb steam;

$Q = +1316.1$ Btu/lb steam

Turbine system 3

$-W = (1020 - 1345.7)$ Btu/lb $= -325.7$ Btu/lb steam;

$W = +325.7$ Btu/lb steam

Condenser system 4

$Q = (28.06 - 1020)$ Btu/lb $= -991.94$ Btu/lb steam

Over-all system

Net heat $= (+1316.1 - 991.94)$ Btu/lb $= +324.16$ Btu/lb steam

Net work $= (+325.7 - 1.5)$ Btu/lb $= +324.2$ Btu/lb steam

Net work = net heat for the cycle.

The efficiency is given by:

$$\eta = \frac{\text{net work output}}{\text{total heat input}} = \frac{324.2 \text{ Btu/lb}}{1316.1 \text{ Btu/lb}} \times 100 = 24.6\%$$

ASSIGNMENT

Closed Systems

1. The internal energy of a system decreases 50 Btu while 150 Btu of work are performed upon the system. What is the direction and magnitude of the heat transfer?

2. A gas expands doing 7000 ft-lb of work while receiving 15 Btu of heat from the surroundings. What is the change in internal energy?

3. A system performing a cyclic operation receives 110 Btu/min and delivers work at the rate of 0.75 hp. How much heat leaves the system per minute and what is the thermal efficiency of the system?

4. Two lb steam at 400 psia and 720 F are contained in an insulated tank. The tank is connected through a small valve to a vertical cylinder fitted with a frictionless weighted piston such that a pressure of 124 psia is required to move it. Initially the piston is at rest at the bottom of the cylinder. The valve is opened and the steam enters the cylinder, forcing the piston upward. When the pressure in the cylinder and tank are equalized, the temperature of the steam in the tank was found to be 440 F. What was the temperature of the steam in the cylinder?

Steady-Flow Systems

5. A fluid enters a nozzle with a velocity of 100 fps and leaves with a velocity of 1500 fps. What is the change in enthalpy of the fluid, neglecting friction heat transfer, etc?

6. A steam jet decreases in velocity from 1500 fps to 750 fps while flowing across a turbine blade. How much work was done per lb of steam by the system of the blade and steam?

7. Steam enters a condenser with an enthalpy of 1000 Btu/lb and a velocity of 500 fps and the condensate leaves at a negligible velocity with an enthalpy of 105 Btu/lb. How much heat is transferred from the fluid?

8. If cooling water entered the condenser in question seven with an enthalpy of 28 Btu/lb and leaves with an enthalpy of 87.9 Btu/lb, how many lb of water are needed per lb of steam condensed?

9. A water pump delivers 100 gal per min at a pressure of 150 psig with a velocity of 50 fps 150 ft above the pump inlet. At the inlet the vacuum is 10 psi, and the velocity is 25 fps. Neglecting changes in internal energy, how many horsepower are required to drive the pump, assuming an efficiency of 80%?

George Howey

Course 1A25
Lesson 1A25.00-3

Heat & Thermodynamics

SOLIDS, LIQUIDS AND VAPOURS

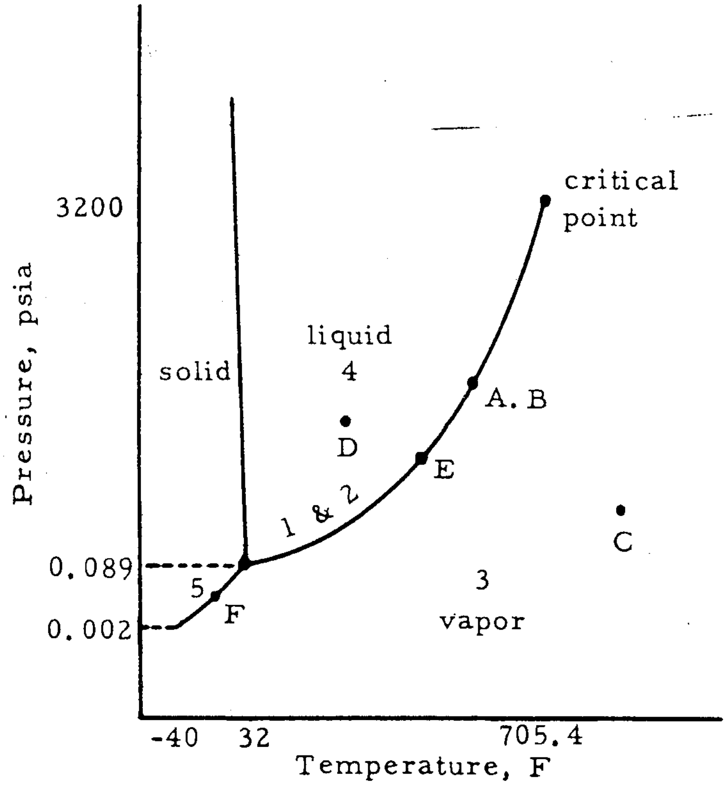
The engineer must possess knowledge of the physical properties of various working fluids before he can solve thermodynamic problems or choose the most advantageous medium for any specific application. In the following discussion emphasis has been placed upon the liquid and phases since these are more frequently encountered by the engineer. The material dealing with the solid phase is presented to complete the phase picture.

Phase and Pressure-Volume Diagrams

The relationship between pressure, specific volume, and temperature of any medium can be represented on a three dimensional surface. Because of the difficulty of visualizing and drawing these surfaces, it is customary to depict the data on projections such as the p - T and p - v planes. Figs. 1 and 2 are examples of these planes for water; other materials have similar diagrams.

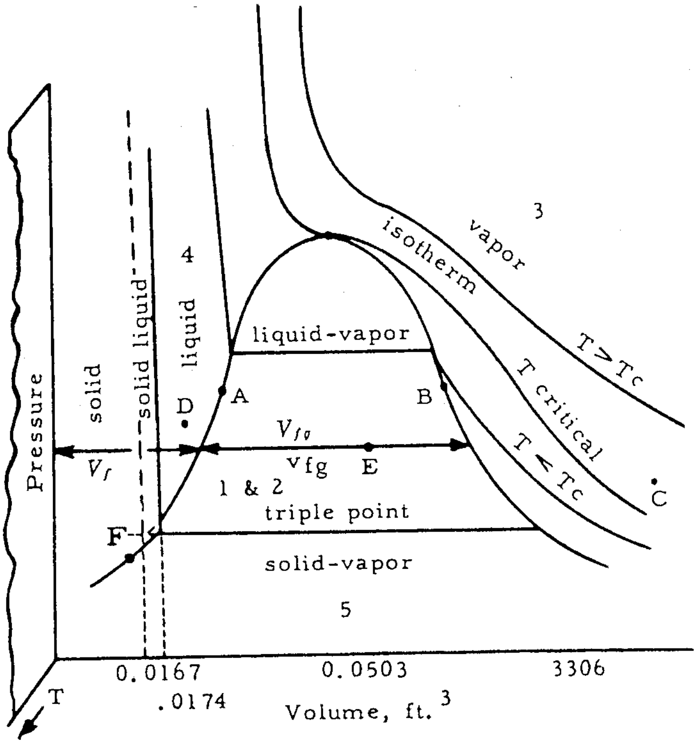
The liquid phase, vapour phase and one solid phase of water are shown in the phase diagram for water in Fig. 1. The diagram is divided into three areas by three lines: the solid-vapour, solid-liquid, and liquid-vapour lines. The triple point is a region in which liquid, solid, and vapour can exist in equilibrium simultaneously. At the critical point there is no distinction between the liquid and vapour phases. It can be seen from Fig. 1 that when ice is heated at constant pressure it will melt into a liquid at a fixed temperature, and upon continual heating will vapourize at another fixed temperature when the pressure is above the triple point and below the critical point. When this process takes place at pressure below the triple point ice sublimates into a vapour without becoming a liquid. Ice, heated at a constant pressure above the critical point, melts into a liquid, and no apparent phase change occurs with continued heating.

It is not possible to present all the p, v, T data on the phase diagram; therefore, the p - v diagram must be used. Since the p - v plane is perpendicular to the p - T plane in the p, v, T coordinate system, the solid-vapour, liquid-vapour and solid-liquid areas of the p - v plane appear as lines on the p - T plane of phase diagram. It will be noted that the solid



Phase diagram for water, not to scale

Fig. 1



Pressure-volume diagram for water not to scale

Fig. 2

area overlaps the liquid area because of the decrease in volume when ice melts into water.

Definition describing the states represented in Figs. 1 and 2 are given below.

Liquid in equilibrium with its vapour is saturated liquid, point A. Vapour in equilibrium with its liquid is saturated vapour, point B. Vapour at a temperature above the saturation temperature at the same pressure is superheated vapour, point C.

Liquid at a lower temperature than the saturation temperature at the same pressure, subcooled liquid, point D. A mixture of vapour and liquid, wet vapour, point E. Solid in equilibrium with its vapour, saturated solid, point F.

The energy given up by a medium when it changes from a liquid to a solid is termed the latent heat of fusion. There is no temperature change.

The latent heat of vapourization is the heat absorbed by a medium in going from the liquid to the vapour phase, and is equal to h_{fg}.

The latent heat of sublimation is the energy absorbed by a medium during the transformation from a solid directly into vapour.

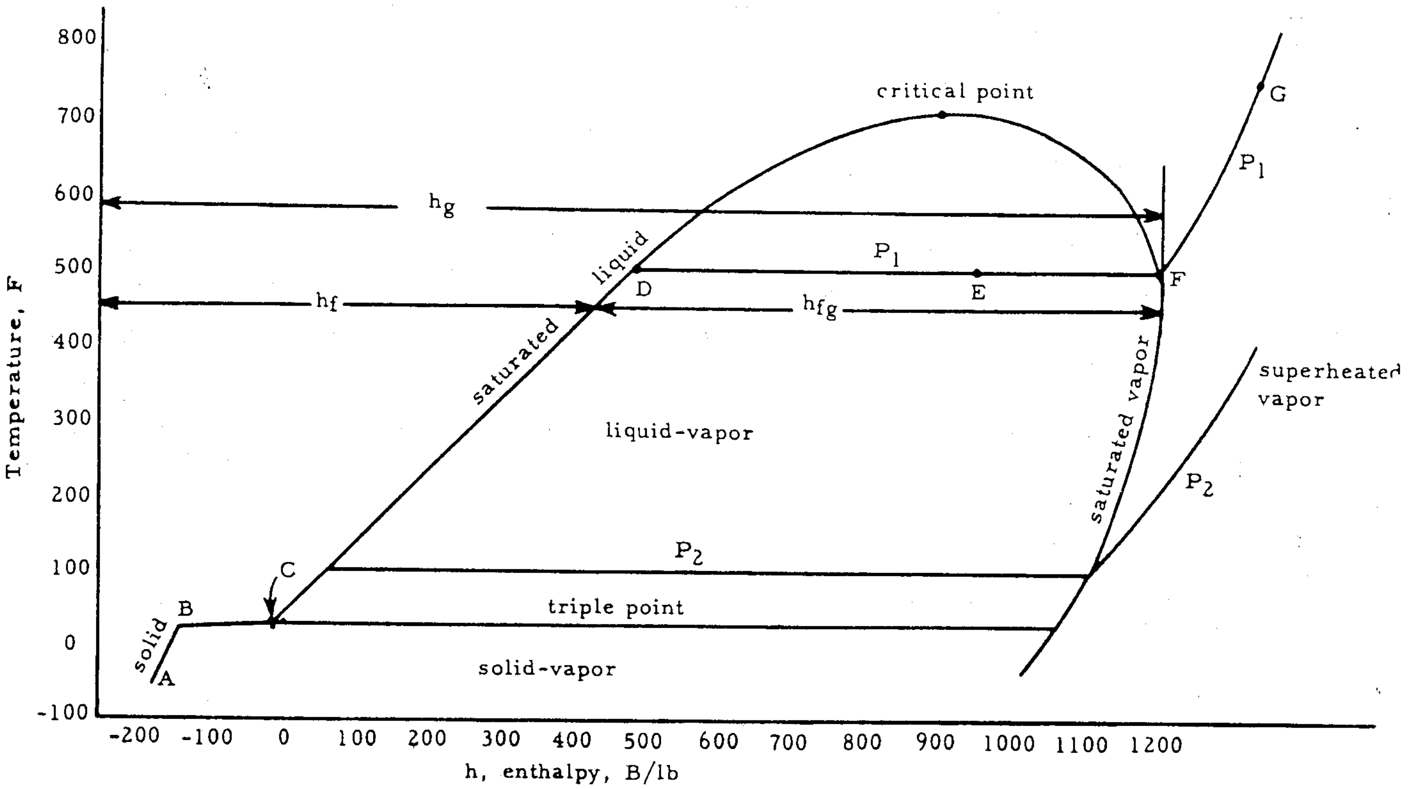
Tabular data for the numbered regions of Fig. 1 and 2 may be found in the correspondingly numbered tables of Thermodynamic Properties of Steam by Keenan and Keyes. These tables will be referred to later as the steam tables.

Temperature-Enthalpy and Temperature-Entropy Diagrams

Consider 1 pound of ice encased in a cylinder fitted with a weighted piston such that the pressure is always constant at p_1. Let the ice be heated from a low temperature (-40 F). The temperature of the ice will rise as the ice absorbs the energy until the melting point (B) is reached. This process is represented on the T - h and T - s diagrams of Figs. 3 and 4 by line A - B. The energy added is represented on the T - h diagram by the change in enthalpy (h) between A and B and by the area under the line A - B on the T - s diagram, since

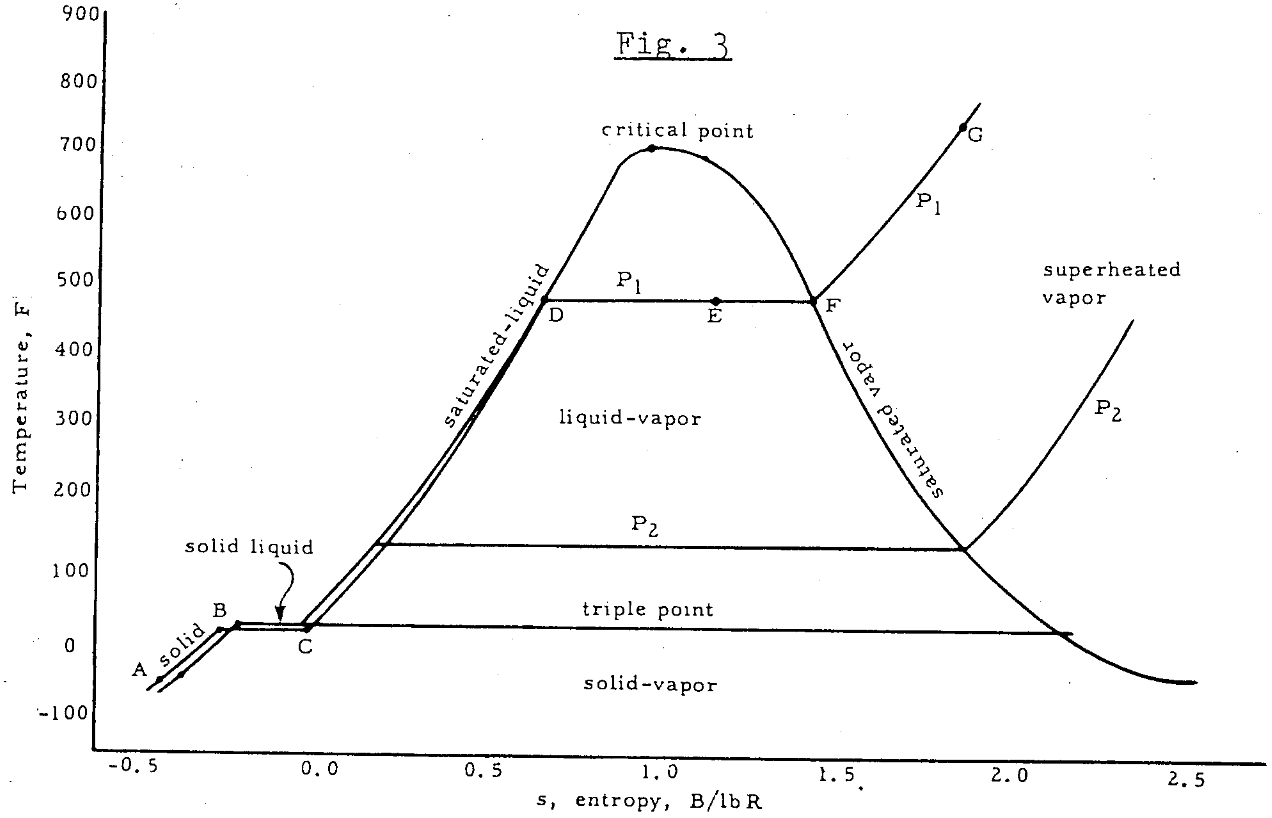
$$Q = \int_R T ds$$

for all reversible processes. As more energy is added to the ice, it will melt at constant pressure and form a subcooled liquid at C. Additional heat causes the liquid to increase in temperature until it reaches the saturation temperature at



Temperature-enthalpy diagram for H_2O, not completely to scale

Fig. 3



Temperature-entropy diagram for H_2O, not completely to scale

Fig. 4

point D. As vapour is formed, the amount of liquid decreases until all liquid disappears and saturated vapour fills the cyclinder, corresponding to point F. At some intermediate point such as E, the mixture of liquid and vapour is referred to as wet vapour having a quality x defined by the equation (1)

$$x = \frac{\text{mass of vapour}}{\text{mass of mixture}} = \frac{\text{weight of vapour}}{\text{weight of mixture}} \quad ....(1)$$

A mixture having a quality of 40 per cent contains 40 per cent vapour and 60 per cent liquid by weight. The values of enthalpies for saturated liquid and vapour are represented by h_f and h_g respectively. The difference between the enthalpies of saturated liquid and saturated vapour at the same pressure (latent heat of vapourization) is designated by h_{fg}. If x = the pounds of vapour per pound of mixture

$$h_x = xh_g + (1 - x)\, h_f \quad ....(2)$$

Since $h_{fg} = h_g - h_f$ by definition

$$h_x = xh_{fg} + xh_f + h_f - xh_f$$

Collecting terms,

$$h_x = h_f + xh_{fg} \quad ....(3)$$

This equation may also be derived by referring to Fig. 3.

In a similar manner equations (4) and (5) may be derived.

$$v_x = v_f + xv_{fg} \quad ....(4)$$

$$s_x = s_f + xs_{fg} \quad ....(5)$$

If the heating process described in the preceding paragraphs is repeated at various pressures, a series of constant pressure lines similar to p_1 may be obtained. The locus of all points B forms the <u>saturated solid curve</u>, while the locus of all D and F points from <u>saturated liquid</u> and <u>saturated vapour curves</u> respectively.

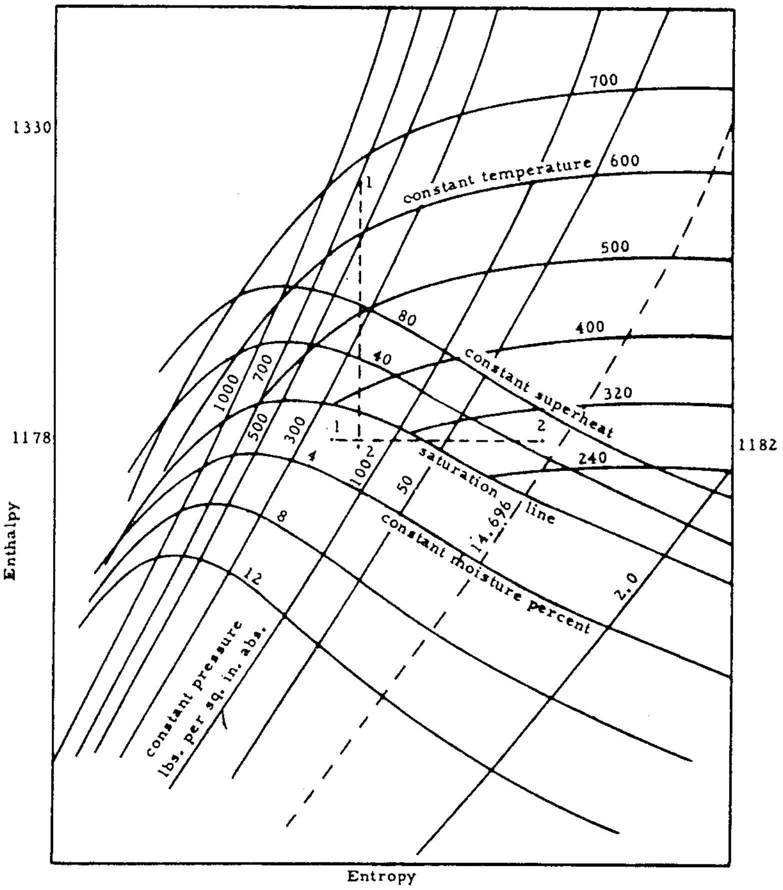
It should be noted that, during a change in phase, both the temperature and pressure are constant.

Vapour Tables

It is customary to present property data in Tables 1 to 4. It should be noted that only the temperature or the pressure needed to fix the state of the saturated liquid, vapour, or solid presented in Tables 1 and 2 while both pressure and temperature are necessary before Tables 3 and 4 can be used. It should be pointed out that Tables 1 to 4 correspond to similarly numbered regions of Figs. 1 and 2.

Special Diagrams

It has been found convenient, in the solution of many problems involving steam, to present the data on an enthalpy-entropy diagram, termed a Mollier diagram after its originator. Usually a small section of the total possible diagram is utilized. Fig. 5 is a skeleton of a Mollier diagram.



Mollier diagram for steam, not completely to scale

Fig. 5

The following examples illustrate the use of the steam tables.

STEAM

Example 1:

What is the boiling point of water at 100 psia pressure?

Solution:

From Table 2 of the steam tables the saturation temperature at 100 psia is 327.81 F, which corresponds to the boiling point.

Example 2:

What is the specific volume, enthalpy, and entropy of saturated steam at 400°F?

Solution:

From Table 1 of the steam tables at 400°F.

v_g = 1.866 ft^3/lb; h_g = 1201 Btu/lb; s_g = 1.5272 Btu/lb R

Example 3:

What are the enthalpy and internal energy of saturated water at 350 psia?

Solution:

From Table 2 of the steam tables at 350 psia

h_f = 409.69 Btu/lb and u_f = 408.45 Btu/lb

Example 4:

What are the specific volume enthalpy and entropy of water at 400 F and 3500 psia?

Solution:

From Table 4 at 3500 psia and 400 F

$$(v - v_f) \times 10^5 = -35.0$$

$$(h - h_f) = +4.01$$

$$(s - s_f) \times 10^3 = -8.21$$

At 400 F,

$$v_f - 0.018639, \quad h_f = 374.97, \quad s_f = 0.56638$$

$$(v - v_f) \times 10^5 = -35, \quad \text{or} \quad v - v_f = \frac{-35}{10^5}$$

Substituting in the value of v_f and solving for v gives:

$$v = 0.018639 - 0.00035 = 0.018289 \text{ ft}^3/\text{lb}$$

$$h - h_f = +4.10$$

Substituting in the value of h_f and solving for h gives:

$$h = 374.97 + 4.1 = 379.07 \text{ Btu/lb}$$

$$s - s_f = \frac{-8.21}{10^3}$$

Substituting in the value of s_f and solving for s gives:

$$s = 0.56638 - 0.00821 = 0.55817 \text{ Btu/lb R}$$

Example 5:

What are the specific volume, enthalpy and entropy of steam having a quality of 90% at 40 psia?

Solution:

From Table 2 of the steam tables at 400 psia

$$v_g = 1.1613; \quad v_f = 0.0193; \quad h_f = 424.0; \quad h_{fg} = 780.5$$

$$s_f = 0.6214; \quad s_{fg} = 0.8630$$

Using equation (4) gives:

$$v = 0.0193 + 0.90 \times 1.1420 = 1.0471 \text{ ft}^3/\text{lb}$$

Using equation (3) gives:

$$h = 424.0 \text{ Btu/lb} + 0.90 \times 780.5 \text{ Btu/lb} = 1126.45 \text{ Btu/lb}$$

Using equation (5) gives:

$$s = 0.6214 \text{ Btu/lb R} + 0.9 \times 0.8630 \text{ Btu/lb R} = 1.3981 \text{ Btu/lb R}$$

Example 6:

Superheated steam at 700 psia and 680 F is expanded at constant entropy to 140 psia. What is the change in enthalpy?

Solution:

On the Mollier diagram, Fig. 5, locate point 1 at the intersection of the 700 psia and the 680 F line. Read h_1 = 1323 Btu/lb. Follow the entropy line downward vertically to the 140 psia line and read h_2 = 1178.

$$\Delta h = 1178 - 1323 = - 145 \text{ Btu/lb}$$

Example 7:

Wet steam at 200 psia is throttled in a calorimeter to a pressure of 20 psia and a temperature of 280 F. What is the quality of the wet steam?

Solution:

Since $h_1 = h_2$ for a throttling process, on the Mollier diagram, locate point 2 at the intersection of the 20 psia line and the 280 F line. Follow horizontally the constant enthalpy line to the left of the 200 psia line, point 1 and read 2% moisture. This gives a quality of 100-2 or 98%. This example may also be solved by using the steam tables. From Table 3 at 20 psia and 280 F obtain h_f = 1182 Btu/lb. From Table 2 at 200 psia obtain h_f = 355.36 Btu/lb and h_{fg} = 843.0 Btu/lb.

Since the enthalpy is constant in a throttling process h_f at 20 psia and 280 F = h wet steam at 200 psia, using equation (4) gives:

$$1182 \text{ Btu/lb} = 355.36 \text{ Btu/lb} + \chi\, 843.0 \text{ Btu/lb}$$

Solving for χ gives:

$$\chi = \frac{1182 - 355.36}{843} \times 100 = 98\%$$

Table 1. Properties of saturated water and steam

Abs. pressure			Specific volume			Enthalpy			Entropy		
Temp. F	psi	In. Hg	Sat. liquid	Evap.	Sat. vapor	Sat. liquid	Evap.	Sat. vapor	Sat. liquid	Evap.	Sat. vapor
t	p		v_f	v_{fg}	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
32	0.08854	0.1803	0.01602	3306	3306	0.00	1075.8	1075.8	0.0000	2.1877	2.1877
50	0.17811	0.3626	0.01603	1703.2	1703.2	18.07	1065.6	1083.7	0.0361	2.0903	2.1264
100	0.9492	1.9325	0.01613	350.3	350.4	67.97	1037.2	1105.2	0.1295	1.8531	1.9826

Table 2. Properties of saturated water and steam

Abs. press psi	Temp. F	Specific volume		Enthalpy			Entropy			Internal energy	
		Sat. liquid	Sat. vapor	Sat. liquid	Evap.	Sat. vapor	Sat. liquid	Evap.	Sat. vapor	Sat. liquid	Sat. vapor
p	t	v_f	v_g	h_f	h_{fg}	h	s_f	s_{fg}	s_g	u_f	u_g
0.20	53.14	0.01603	1526.0	21.21	1063.8	1085	0.0422	2.0741	2.1163	21.21	1007.3
1.00	101.74	0.01614	333.6	69.70	1036.3	1106.0	0 1326	1.8456	1.9782	69.70	974.6
14.696	212.0	0.01672	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	180.02	897.5
100.0	327.81	0.01744	4.432	298.4	888.8	1187.2	0.4740	1.1286	1.6026	298.08	807.1

Table 3. Properties of superheated vapor

Abs. press. psi (sat. temp.)		Sat. liquid	Sat. vapor	Temperature F						
				300	400	600	800	1000	1300	1600
	v	0.02	333.6	452.3	512.0	631.2	750.4	869.5	1048.3	1227.0
1	h	69.7	1106.0	1195.8	1241.7	1335.7	1432.8	1533.5	1691.2	1857.5
(101.74)	s	0.1749	1.920	2.0387	2.0955	2.1938	2.2778	2.3519	2.4501	2.5373
10	v	0.02	38.42	45.00	51.04	63.03	74.98	86.92	104.8	122.69
(193.21)	h	161.2	1143.3	1193.9	1240.6	1335.1	1432.5	1533.2	1691.1	1857.3
	s	0.2835	1.7876	1.8595	1.9172	2.0160	2.1002	2.1744	2.2727	2.3598

Table 4. Properties of compressed liquid

Abs. press. psi (sat. temp.)	Saturated liquid	Temperature F		
		32	200	400
600	$(v-v_f) \times 10^5$	−3.5	−3.4	−4.4
	$(h-h_f)$	+1.80	+1.31	+0.39
(486.2)	$(s-s_f) \times 10^3$	+0.07	−0.74	−0.96

Table [illegible]-5. Properties of saturated solid and vapor

	Specific volume			Enthalpy			Entropy			Internal energy		
Temp. F	Abs. press psi	Sat. solid	Sat. vapor	Sat. solid	Subl.	Sat. vapor	Sat. solid	Subl.	Sat. vapor	Sat. solid	Subl.	Sat. vapor
t	p	v_i	v_g 10^{-3}	h_i	h_{ig}	h_g	s_i	s_{ig}	s_g	u_i	u_{ig}	u_g
32	0.0885	0.01747	3.306	−143.35	1219.1	1075.8	−0.2916	2.4793	2.1877	−143.35	1164.9	1021.6
15	0.0396	0.01745	4.508	−151.75	1220.2	1068.4	−0.3089	2.5698	2.2609	−151.75	1167.9	1016.1
5	0.0240	0.01743	11.53	−156.56	1220.6	1064.0	−0.3192	2.6260	2.3068	−156.56	1169.4	1012.8
0	0.0185	0.01742	14.77	−158.93	1220.7	1061.8	−0.3241	2.6546	2.3305	−158.93	1170.0	1011.1
−10	0.0108	0.01741	24.67	−163.59	1221.0	1057.4	−0.3346	2.7143	2.3797	−163.59	1171.4	1007.8

ASSIGNMENT

1. What is the change in specific volume when one pound of ice at 32 F in converted to steam at 1 psia pressure and 800 F? What is the change in internal energy?

2. Wet steam at 100 psia having a quality of 10% is converted to dry saturated steam at constant pressure in a heat exchanger. How much heat is required per 10 pounds of steam?

3. What is the internal energy of steam at 14.7 psia having a quality of 70%?

4. Steam at 10 psia and 300 F is superheated at constant pressure to 1600 F.

   (a) How much heat is required per pound of steam?
   (b) What is the change in enthalpy?
   (c) What is the change in internal energy?

5. Water at 100 F is converted to dry saturated steam at constant temperature. What is the change in volume?

6. One pound of superheated steam at 10 psia and 800 F is cooled at constant volume until the pressure is 1 psia.

   (a) What is the final quality of the steam?
   (b) How much heat must be removed?

George Howey

Heat & Thermodynamics

PERFECT GAS

Ideal Equations of State

The results of certain experiments at relatively low pressure, led Robert Boyle to formulate his well-known law, which which states that the pressure of a gas expanding at constant temperature varies universely as the volume, or

$$p_1v_1 = p_2v_2 = p_3v_3 = \text{constant} \quad \ldots\ldots\ldots\ldots(1)$$

Charles, likewise as the result of experimentation, concluded that the pressure of a gas varies directly with temperature when the volume is held constant, and the volume varies directly with temperature when the pressure is held constant, or

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

By combining the results of Charles and Boyles experiments the relationship

$$\frac{pv}{T} = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

may be obtained.

The constant of equation (4) is called the ideal gas constant and is designated by R; thus, the ideal gas equation of state becomes:

$$pv = RT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

(Note that the pressure and temperature are absolute values).

This equation may also be derived from the kinetic theory of gases. The values of the ideal gas constant R for several of the more common gases are given in Table 1.

TABLE 1

Gas	Chemical symbol	Molecular weight	Gas Constant ft-lb/lb R	Specific heat Btu/lb		Specific heat ratio
		M	R	c_v	c_p	k
Air		28.95	53.35	0.172	0.240	1.40
Carbon dioxide	CO_2	44.00	35.13	0.160	0.205	1.28
Hydrogen	H_2	2.016	766.80	2.44	3.42	1.40
Nitrogen	N_2	28.02	55.16	0.176	0.247	1.40
Oxygen	O_2	32.0	48.31	0.155	0.217	1.40
Steam	H_2O	18.016	85.81	0.36	0.46	1.28

The ideal gas equation for w pounds of gas occupying a total volume V becomes

$$pV = wRT \qquad \text{.................................}(6)$$

When the quantity of gas is expressed in pound mole units (one pound mole being a quantity of gas equivalent to the molecular weight of the gas expressed in pounds), equation (6) is written

$$pV = nR_oT \qquad \text{.................................}(7)$$

where n is the number of pound moles and R_o is the universal gas constant. The value of R_o, expressed in the same set of units, is the same for all gases because of Avogadro's law, which may be stated: equal volume of gases under the same conditions contain the same number of molecules.

The individual gas constant R may be obtained by dividing the universal gas constant R_o by the molecular weight of the gas:

$$R = \frac{R_o}{M} \qquad \text{.................................}(8)$$

When the pressure is expressed in pounds per square foot, the volume in cubic feet, the temperature in degrees Rankine, and the quantity of gas in pound moles, R_o is equal to approximately

$$\frac{1544 \text{ ft-lb}}{\text{lb mole R}}$$

Example 1:

Determine the gas constant for carbon dioxide from the universal gas constant.

Solution:

Substituting 44, the molecular weight of CO_2 into equation (8) gives

$$R = \frac{R_o}{44} = 1544 \text{ ft-lb/lb mole R} \times \frac{1}{44 \text{ lb/lb mole}}$$

$$= 35.09 \text{ ft-lb/lb R}$$

The units of R must always be consistent with the units of pressure, volume, and temperature employed in the gas equation.

No real gas follows the ideal gas law or equation completely; however, most gases are in approximate agreement at low pressures (relative to their critical pressure), provided the temperatures are sufficiently high.

Example 2:

Determine the pressure exerted by 5 lb air enclosed in a 20 ft^3 tank at 140 F.

Solution:

METHOD A   From Table 1, R = 53.35

$$\text{Specific volume } v = \frac{20 \text{ ft}^3/\text{lb}}{5} = 4.0 \text{ ft}^3/\text{lb}$$

$$T = 140 + 460 = 600 \text{ R}$$

By equation (5)

$$p \times 4.0 \text{ ft}^3/\text{lb} = 53.35 \text{ ft-lb/lb R} \times 600 \text{ R}$$

$$p = \left(\frac{53.35 \times 600}{4}\right) \text{psf} = 8000 \text{ psf} = \frac{8000}{144} = 55.5 \text{ psia}$$

METHOD B

The weight of gas w = 5 lb, temperature T = 600 R, and total volume V = 20 ft^3. By equation (6)

$$p \times 20 \text{ ft}^3 = 5 \text{ lb} \times 53.35 \text{ ft-lb/lb R} \times 600 \text{ R}$$

$$p = \left(\frac{5 \times 53.35 \times 600}{20}\right) \text{psf} = 8000 \text{ psf} = 55.5 \text{ psia}$$

METHOD C

The number of lb moles of gas are

$$n = \frac{\text{number of pounds}}{\text{molecular weight}} = \frac{5}{28.95} = 0.1727$$

Using equation (7) gives

$$p \times 20 \text{ ft}^3 = 0.1727 \text{ lb mole} \times 1544 \text{ ft-lb/lb mole R} \times 600 \text{ R}$$

$$p = \frac{0.1727 \times 1544 \times 600}{20} \text{psf} = 8000 \text{ psf} = 55.5 \text{ psia}$$

Example 3:

Determine the volume of a tank required to hold 50 lb carbon dioxide at 220 F and 23.3 psig pressure when the atmospheric pressure is 14.7 psia.

Solution:

The pressure used in the ideal gas equation must be absolute pressure, thus

$$p = 23.3 + 14.7 = 38.0 \text{ psia}$$

$$p = 38 \text{ lb/in}^2 \times 144 \text{ in}^2/\text{ft}^2 = 5470 \text{ psf};$$

$$T = 220 + 460 = 680 \text{ R}$$

From Table 1, R = 35.09 ft-lb/lb R

By equation (6)

$$5470/\text{ft}^2 \times V = 50 \times 35.09/\text{lb R} \times 680 \text{ R}$$

$$V = \frac{50 \times 35.09 \times 680}{5470} = 218 \text{ ft}^3$$

Specific Heats

The specific heats of all gases vary with temperature. Table 1 contains the specific heat values for several common gases.

The kinetic theory of gases may be used to obtain the molar heats for monatomic and diatomic gases.

Monatomic gases	$C_v = 3$ Btu/lb mole R
	$C_p = 5$ Btu/lb mole R
Diatomic gases	$C_v = 5$ Btu/lb mole R
	$C_p = 7$ Btu/lb mole R

Example 4:

Determine the constant volume specific heat of hydrogen using the kinetic theory value and compare it with the value found in Table 1.

Solution:

The molecular weight of hydrogen (a diatomic gas) is 2. Therefore, the constant volume specific heat is equal to the molecular heat divided by 2.

$$c_v = \frac{C_v}{2} = \frac{5 \text{ Btu}}{\text{lb mole R}} \times \frac{1}{2 \text{ lb/lb mole}} = 2.50 \text{ Btu/lb}$$

The value of c_v from Table 1 is 2.44; thus, the kinetic theory value deviates

$$\frac{0.06}{2.44} \times 100 = 2.5\% \text{ from the experimental value.}$$

A polytropic specific heat has been introduced at various times for convenience in heating the polytropic process $p_1V_1{}^n = p_2V_2{}^n$. This is not a true specific heat since it does not conform to the definition of constant volume or constant pressure specific heats. The polytropic specific heat may have a negative value, indicating a decrease in temperature during heat absorption.

$$c_n = \frac{R\ (k - n)}{J\ (k - 1)(1 - n)} \quad \text{or} \quad \frac{c_v\ (k - n)}{(1 - n)}$$

Internal Energy, Enthalpy, and Specific Heat Relationships

The results of experimentation with gases flowing from one chamber into an evacuated chamber, without doing external work and in the absence of any observable heat transfer, led Joule to conclude that the internal energy of an ideal gas depends upon the temperature and is independent of the volume. Stated mathematically Joule's law for an ideal gas is:

$$u = f(T) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Since internal energy is a property, it is independent of the process; therefore, any process may be used to evaluate it. Equation 2 lesson 2 written for 1 pound of a perfect gas undergoing a constant volume process becomes:

$$Q = u_2 - u_1$$

There is no external work since V = constant, but $Q = c_v (T_2 - T_1)$ for a constant volume process; therefore

$$c_v (T_2 - T_1) = u_2 - u_1 \quad \text{..................(10)}$$

For an infinitely small change in temperature,

$$du = c_v dT \quad \text{..............................(11)}$$

Example 5:

Determine the internal energy change when 50 lb of CO_2 are heated from 50 F to 250 F assuming constant specific heat.

Solution:

From Table 1, c_v for CO_2 = 0.160 Btu/lb F

Using equation (10) for 1 lb of gas gives

$$u_2 - u_1 = \frac{0.160 \text{ Btu}}{\text{lb F}} (250 - 50) \text{ F} = 32 \text{ Btu/lb}$$

For 50 lb

$$U_2 - U_1 = \frac{32 \text{ Btu}}{\text{lb}} \times 50 \text{ lb} = 1600 \text{ Btu}$$

It can be proven mathematically that the enthalpy of an ideal gas is evaluated by equation (12)

$$dh = c_p dT \quad \text{..................................(12)}$$

When a constant specific heat is assumed, the equation becomes

$$h_2 - h_1 = c_p (T_2 - T_1) \quad \text{......................(13)}$$

Example 6:

Determine the change in enthalpy when 100 lb air are cooled from 400 F to 120 F, assuming a constant specific heat.

Solution:

The value of c_p for air from Table 1 is 0.24 Btu/lb F.

Solution: (cont'd)

Using equation (13) for lb air gives

$$h_2 - h_1 = 0.24 \text{ Btu/lb F} \times (120 - 400) \text{ F} = -67.2 \text{ Btu/lb}$$

for 100 lb

$$H_2 - H_1 = -67.2 \text{ Btu/lb} \times 100 \text{ lb} = -6720 \text{ Btu}$$

Note that the answer is negative indicating a reduction in enthalpy.

Example 7:

Determine the relationship between c_p, c_v and R for an ideal gas.

Solution:

Write the first law of thermodynamics for a reversible non-flow constant pressure process, using 1 lb of an ideal gas as the system.

$$Q = u_2 - u_1 + \frac{W}{J}$$

$$Q = c_p (T_2 - T_1); \quad u_2 - u_1 = c_v (T_2 - T_1)$$

and

$$W = \frac{p(v_2 - v_1)}{J} = \frac{p_2 v_2}{J} - \frac{p_1 v_1}{J} = \frac{R T_2}{J} - \frac{R T_1}{J} = \frac{R}{J}(T_2 - T_1)$$

Therefore

$$c_p (T_2 - T_1) = c_v (T_2 - T_1) + \frac{R}{J}(T_2 - T_1)$$

or

$$c_p = c_v + \frac{R}{J} \qquad \text{......................}(14)$$

The ratio of specific heats is commonly designated by k:

$$k = \frac{c_p}{c_v} \qquad \text{......................}(15)$$

Solving this relationship for c_v and substituting it in equation (14) gives:

$$c_p = \left(\frac{k}{k - 1}\right) \frac{R}{J} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

General Perfect Gas Equations

Given in Table 2 are the general nonflow reversible process equations for an ideal gas. All equations are given for 1 pound of gas; therefore, total values are obtained by multiplying these equations by the number of pounds of gas involved.

Reversible Nonflow Process Equation

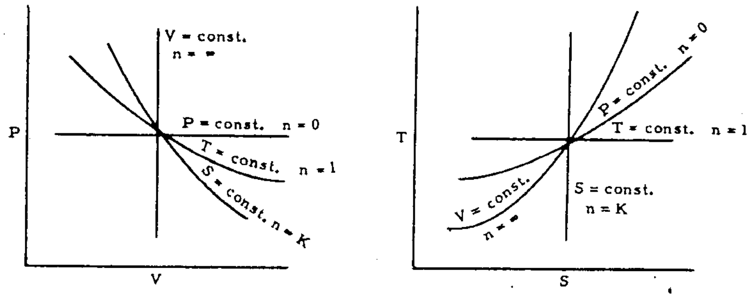
It is possible to express the equation of the reversible ideal gas nonflow process plotted on the p - v plane by the equation:

$$pv^n = \text{constant} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

The value of n changes with the process.

The most common reversible gas processes are: (1) isobaric or constant pressure; (2) isovolumic or constant volume; (3) isothermal or constant temperature; (4) isentropic, constant entropy or reversible adiabatic; and (5) polytropic for which n of equation (24) remains n; whereas n has some definite value for all of the other changes. The general equations of Table 2 applied to these specific processes reduce to several simple equations when constant specific heats are assumed. Table 3 contains the more common forms of these equations.

It is often convenient when analyzing one or more processes to picture them on the p - v, T - s, h - s, etc. planes as shown in Fig. 1.



Reversible processes for an ideal gas presented upon the p - v and T - s diagrams

Fig. 1

TABLE 2

TERM	DIFFERENTIAL EXPRESSION	TOTAL EXPRESSION FOR 1 POUND OF GAS	
Work	δW	$W = \int_{v_1}^{v_2} p\,dv$	................(17)
Heat	$\delta Q = c\,dT$ (c depends upon process)	$Q = \int_{T_1}^{T_2} c\,dT$	...............(18)
Change in internal energy	$du = c_v\,dT$	$u_2 - u_1 = \int_{T_1}^{T_2} c_v\,dT$	.......(19)
Change in enthalpy	$dh = c_p dT$	$h_2 - h_1 = \int_{T_1}^{T_2} c_p dT$	.......(20)
Change in entropy	$ds = c_v \frac{dT}{T} + \frac{R}{J}\frac{dv}{v}$	$s_2 - s_1 = \int_{T_1}^{T_2} c_v \frac{dT}{T} + \frac{R}{J} \int_{v_1}^{v_2} \frac{dv}{v}$	.(21)
	$ds = c_p \frac{dT}{T} - \frac{R}{J}\frac{dp}{p}$	$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \frac{R}{J} \int_{p_1}^{p_2} \frac{dp}{p}$	(22)
	$ds = c_p \frac{d_v}{v} + c_v \frac{dp}{p}$	$s_2 - s_1 = \int_{v_1}^{v_2} c_p \frac{dv}{v} + \int_{p_1}^{p_2} c_v \frac{dp}{p}$	.(23)

TABLE 3

Equations for reversible nonflow ideal gas processes, assuming a constant specific heat.

Process	Isobaric p = constant	Isovolumic V = constant	Isothermal T = constant	Isentropic S = constant	Polytropic pV^n = constant
Specific heat c	$c_p = \frac{kR}{(k-1)J}$	$c_v = \frac{R}{(k-1)J}$		0	$c_n = c_v \frac{(k-n)}{(1-n)}$
Exponent n for pV^n = const.	0		1	k	n = any value
Quantity of heat $_1Q_2$	$wc_p\,(T_2 - T_1)$	$wc_v\,(T_2 - T_1)$	$\frac{p_1V_1}{J} \ln \frac{V_2}{V_1}$ $\frac{wRT}{J} \ln \frac{V_2}{V_1}$ $\frac{wRT}{J} \ln \frac{p_1}{p_2}$	0	$wc_n\,(T_2 - T_1)$
Quantity of work $_1W_2$	$p(V_2 - V_1)$	0	$p_1V_1 \ln \frac{V_2}{V_1}$ $wRT \ln \frac{V_2}{V_1}$ $wRT \ln \frac{p_1}{p_2}$	$\frac{p_2V_2 - p_1V_1}{1-k}$ $wc_v\,(T_2 - T_1)$	$\frac{p_2V_2 - p_1V_1}{1-n}$ $\frac{wR\,(T_2 - T_1)}{1-n}$
$U_2 - U_1$	$wc_v\,(T_2 - T_1)$ $\frac{p_2V_2 - p_1V_1}{(k-1)J}$	$wc_v\,(T_2 - T_1)$ $\frac{p_2V_2 - p_1V_1}{(k-1)\,J}$	0	$wc_v\,(T_2 - T_1)$ $\frac{p_2V_2 - p_1V_1}{(k-1)J}$	$wc_v\,(T_2 - T_1)$ $\frac{p_2V_2 - p_1V_1}{(k-1)J}$
$H_2 - H_1$	$wc_p\,(T_2 - T_1)$ $\frac{k(p_2V_2 - p_1V_1)}{(k-1)J}$	$wc_p\,(T_2 - T_1)$ $\frac{k(p_2V_2 - p_1V_1)}{(k-1)J}$	0	$wc_p\,(T_2 - T_1)$ $\frac{k(p_2V_2 - p_1V_1)}{(k-1)J}$	$wc_p\,(T_2 - T_1)$ $\frac{k(p_2V_2 - p_1V_1)}{(k-1)J}$
$S_2 - S_1$	$wc_p \ln \frac{T_2}{T_1}$ $wc_p \ln \frac{V_2}{V_1}$	$wc_v \ln \frac{T_2}{T_1}$ $wc_v \ln \frac{p_2}{p_1}$	$wR \ln \frac{V_2}{V_1}$ $wR \ln \frac{p_1}{p_2}$	0	$wc_n \ln \frac{T_2}{T_1}$
$p\ V\ T$	$\frac{V_1}{V_2} = \frac{T_1}{T_2}$ $p_1 = p_2$	$\frac{p_1}{p_2} = \frac{T_1}{T_2}$ $V_1 = V_2$	$p_1V_1 = p_2V_2$ $T_1 = T_2$	$p_1V_1{}^k = p_2V_2{}^k$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$	$p_1V_1{}^n = p_2V_2{}^n$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$

The p - v diagram is of special interest, since the area under the reversible process curve represents the work performed during the nonflow process.

The T - s diagram is likewise of importance since the area under the reversible process curve represents the heat transfer during the process. The areas cannot be negative; therefore, the direction of the process must be considered when determining the sign of the work performed or heat transferred.

Example 8:

Determine the work performed, heat absorbed, and changes in internal energy, enthalpy and entropy when 75 lb air are heated or expanded at constant pressure from 50 psia and 60 F to 250 F during a nonflow process.

Solution:

As the first step in the solution of this problem, the specific volume will be determined at the initial point by using the ideal gas equation of state.

$$v = \frac{53.35 \text{ ft-lb}}{16 \text{ R}} \times \frac{(460 + 60) \text{ R}}{50\frac{\text{lb}}{\text{in}^2} \times 144 \text{ in}^2/\text{ft}^2} = 3.85 \text{ ft}^3/\text{lb}$$

All necessary equations will be obtain from the constant pressure process column of Table 3; however, the general equations may be used if desired.

$$\frac{v_2}{v_1} = \frac{T_2}{T_1} \qquad v_2 = 3.85 \frac{(460 + 250)}{(460 + 60)} = 5.25 \text{ ft}^3/\text{lb}$$

$$\text{Work/lb} = p(v_2 - v_1) = 50 \times 144\frac{\text{lb}}{\text{ft}^2} (5.25 - 3.85) \text{ ft}^3/\text{lb}$$

$$= 10{,}100 \text{ ft-lb/lb}$$

$$\text{Total work} = 10{,}100 \text{ ft-lb/lb} \times 75 \text{ lb} = 756{,}000 \text{ ft-lb}$$

$$\text{Heat absorbed/lb} = Q = c_p (T_2 - T_1) = 0.24 \text{ Btu/lb F} \times (250 - 60)$$

$$= 45.6 \text{ Btu/lb}$$

Total heat absorbed = 75 lb x 45.6 Btu/lb = 3420 Btu

Change in internal energy/lb

$$= u_2 - u_1 = c_v\ (T_2 - T_1) = 0.172 \text{ Btu/lb F} \times (250 - 60)\text{F}$$

$$= 32.7 \text{ Btu/lb}$$

Total internal energy change = 75 lb x 32.7 Btu/lb

= 2450 Btu

These three values may be checked by using the first law of thermodynamics

$$Q = U_2 - U_1 + \text{work}$$

Thus

$$3420 = 2450 \text{ Btu} + \frac{756{,}000 \text{ ft-lb}}{778 \text{ ft-lb/Btu}}$$

$$3420 \text{ Btu} = 3425 \text{ Btu}$$

The difference is the result of slide rule calculations.

Change in enthalpy/lb = $h_2 - h_1 = c_p\ (T_2 - T_1)$

$$= 0.240 \text{ Btu/lb R} \times (710 - 520) \text{ R}$$

$$= 45.6 \text{ Btu/lb}$$

Total change in enthalpy = 75 lb x 45.6 Btu/lb = 3420 Btu

Change in entropy/lb = $s_2 - s_1 = c_p \ln \dfrac{T_2}{T_1}$

$$= 0.240 \text{ Btu/lb R} \times \ln \frac{710}{520}$$

$$= 0.075 \text{ Btu/lb R}$$

Total change in entropy = 75 lb x 0.075 Btu/lb R = 5.62 Btu/R

Example 9:

During a reversible nonflow polytropic process for which $n = 1.25$, 10 lb air are compressed from a pressure of 14.7 psia and a temperature of 60 F to a pressure of 75 psia. Determine (a) the work of compression; (b) the heat transfer; (c) the change in internal energy; (d) the change in enthalpy; and (e) the change in entropy.

Solution:

The initial specific volume as determined by the Gas Laws are

$$v = 53.35 \text{ ft-lb/lb} \times \frac{(460 + 60)}{14.7 \times 144 \frac{\text{lb}}{\text{ft}^2}} = 13.1 \text{ ft}^3\text{/lb}$$

The final volume by the process equations is:

$$p_1 v_1{}^{1.25} = p_2 v_2{}^{1.25} \quad \text{or } v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.25}}$$

$$v_2 = \left(\frac{14.7}{75}\right)^{\frac{1}{1.25}} \times 13.1 \text{ ft}^3\text{/lb} = 3.56 \text{ ft}^3\text{/lb}$$

(a) The work is equal to $\int$ pdv; thus:

$$W = w \int_{13.1}^{3.56} pdv, \quad \text{but} \quad p_v{}^{1.25} = \text{constant}, \quad \text{or} \quad p = \frac{p_1 v_1{}^{1.25}}{v^{1.25}}$$

$$W = wp_1 v_1{}^{1.25} \int_{13.1}^{3.56} \frac{dv}{v^{1.25}} = \frac{-wp_1 v_1{}^{1.25}}{0.25} \left[\frac{1}{v_2{}^{0.25}} - \frac{1}{v_1{}^{0.25}}\right]_{13.1}^{3.56}$$

$$W = \frac{-10 \text{ lb} \times (14.7 \times 144)}{0.25} \frac{\text{lb}}{\text{ft}^2} \times (13.1 \text{ ft}^3\text{/lb})^{1.25}$$

$$\left[\frac{1}{(356)^{0.25}} - \frac{1}{(13.1)^{0.25}}\right] = -429{,}000 \text{ ft-lb}$$

This answer may be checked by using the equation for work from Table 3:

$$W = \frac{w(p_2v_2 - p_1v_1)}{1 - n} =$$

$$\frac{10\text{ lb}\left[(75 \times 144)\frac{\text{lb}}{\text{ft}^2} \times 3.56\text{ ft}^3/\text{lb} - (14.7 \times 144)\frac{\text{lb}}{\text{ft}^2} \times \frac{13.1\text{ ft}^3}{\text{lb}}\right]}{1 - 1.25}$$

$$= -\ 429{,}000\text{ ft-lb}$$

The negative sign indicates that work is performed upon the air.

(b) The heat transfer may be obtained by using the first law:

$$Q = U_2 - U_1 + W = 346\text{ Btu} - \frac{429{,}000\text{ ft-lb}}{778\text{ ft-lb/Btu}} = -\ 207\text{ Btu}$$

Two hundred Btu left the air. This value may be checked by using the polytropic specific heat.

$$c_n = 0.173\text{ Btu/lb R}\left(\frac{1.40 - 1.25}{1 - 1.25}\right) = -\ 0.1037\text{ Btu/lb R}$$

The negative sign indicates heat removed with temperature increase.

$$Q = wc_n\ (T_2 - T_1) = 10\text{ lb}\ (-0.1037\text{ Btu/lb R})(720 - 520)\text{R}$$

$$= -\ 208\text{ Btu}$$

(c) The final temperature by the equation from Table 2 is:

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{1.25 - 1} = (520\text{ R})\left(\frac{13.1}{3.56}\right)^{0.25} = 720\text{ R}$$

The change in internal energy is:

$$U_2 - U_1 = wc_v \ (T_2 - T_1) = 10 \text{ lb} \times 0.173 \text{ Btu/lb R } (720 - 520) \text{ R}$$

$$= + 346 \text{ Btu}$$

(d) The change in enthalpy is:

$$H_2 - H_1 = wc_p \ (T_2 - T_1) = 10 \text{ lb} \times 0.24 \text{ Btu/lb R} \times (720 - 520) \text{ R}$$

$$= + 480 \text{ Btu}$$

(e) The change in entropy is obtained by using the polytropic equation from Table 3, thus:

$$S_2 - S_1 = wc_n \ln \frac{T_2}{T_1} = 10 \text{ lb } (-0.1037) \text{ Btu/lb R} \ln \frac{720}{520}$$

$$= - 0.336 \text{ Btu/R}$$

Gas Tables

To make this table air is assumed to obey the ideal gas law and each gas has a specific heat that varies with temperature. A reference state is assumed, and any other state is assumed to be reached by an isentropic process from the reference state. This procedure permits the internal energy, enthalpy, pressure and volume changes to be written in terms of the temperatures of the reference and second states.

By writing equation (22) for the isentropic process it can be shown that

$$\ln \frac{p}{p_o} = \frac{J}{R} \int_{T_o}^{T} c_p \frac{dT}{T} = \ln p_r$$

$$\text{where } p_r = \frac{p}{p_o}$$

Equation (20) written in terms of T and T_o becomes:

TABLE 4

Air at low pressure (for one pound)

T	t	h	p_r	u	v_r	φ
100	- 359.7	23.74	0.003841	16.88	9643	0.19714
125	- 334.7	29.72	0.008372	21.15	5531	0.25054
150	- 309.7	35.71	0.015824	25.43	3512	0.29418
200	- 259.7	47.67	0.04320	33.96	1714.9	0.36303
300	- 159.7	71.61	0.17795	51.04	624.5	0.46007
500	40.3	119.48	1.0590	85.20	174.90	0.58233
800	340.3	191.81	5.526	136.97	53.63	0.69558
1000	540.3	240.98	12.298	172.43	30.12	0.75042
2000	1540.3	504.71	174.00	367.61	4.258	0.93205
4000	3540.3	1088.26	3280	814.06	0.4518	1.13334
6500	6040.3	1858.44	28974	1412.87	0.08310	1.28268

$$h - h_o = \int_{T_o}^{T} c_p \, dT$$

The value of u may be obtained from the equation:

$$u = h - pv = h - RT$$

By letting

$$\varphi = \int_{T_o}^{T} c_p \frac{dT}{T},$$ equation (22) can be written:

$$s - s_o = \varphi - \frac{R}{J} \ln p_r$$

or

$$s_2 - s_1 = \phi_2 - \phi_1 - \frac{R}{J} \ln \frac{p_2}{p_1}$$

The relative volume v_r may be obtained from relationship

$$v_r = \frac{RT}{p_r}$$

Once T_o is established, values of p_r, u, h, N_2 and ϕ become functions of T alone.

Example 10:

Determine the work required to compress 1 lb air in steady flow from 23 psia and 40 F to 120 psia by a compressor having an adiabatic efficiency of (a) 100% and (b) 87%. Determine the temperature of the air leaving the compressor and the change in entropy.

Solution:

(a) The work for this process is obtained by applying the steady flow energy equation. From Table 4 at 500 R, $h_1 = 119.48$, $p_{r1} = 1.059$, $\phi = 0.58233$

Now
$$\frac{p_2}{p_1} = \frac{p_2/p_o}{p_1/p_o} = \frac{p_{r2}}{p_{r1}}$$

Therefore
$$p_{r2} = \frac{p_2}{p_1} \times p_{r1}$$

or
$$p_{r2} = 1.059 \times \frac{120}{23} = 5.526$$

From Table 4 $p_{r2} = 5.526$, $h_2 = 191.81$, $T_2 = 800$ R and

$\phi_2 = 0.69558$

The value for this work may be obtained by writing equation (4) lesson 2 for this reversible adiabatic process, neglecting kinetic and potential energy

$$W = h_2 - h_1 = 191.81 - 119.48 = 72.33 \text{ Btu/lb air}$$

By the entropy change equation:

$$s_2 - s_1 = 0.69558 - 0.58233 - \frac{53.35}{778} \ln \frac{120}{23} = 0.11325 - 0.1132 = 0$$

(b) By definition adiabatic efficiency is the ratio of the ideal isentropic work of compression to the actual adiabatic work of compression:

$$\eta = \frac{\Delta h \text{ ideal}}{\Delta h \text{ actual}}, \text{ or } \Delta h \text{ actual} = \frac{\Delta h \text{ ideal}}{\eta} = \frac{72.33 \text{ Btu/lb}}{0.87}$$

$$= 83.2 \text{ Btu/lb}$$

$$h_2 = h_1 + \Delta h = 119.48 + 83.2 = 202.68 \text{ Btu/lb}$$

From Table 4 at h = 202.68, T_2 = 844.5 R, and φ = 0.70878

$$s_2 - s_1 = 0.70878 - 0.58233 - \frac{53.35}{778} \ln \frac{120}{23} = 0.0133 \text{ Btu/R.}$$

ASSIGNMENT

1. During a reversible nonflow compression for which $p = (5V + 15)$ psia, the volume of a gas changes from 4 to 1.5 ft^3. What is the work of compression?

2. Three ft^3 of air at 60 F and 45 psig are heated at constant pressure to 120 F. (a) How much heat was transferred? (b) What was the change in internal energy? (c) How much work was performed?

3. Fifteen ft^3 of hydrogen at a pressure of 250 psia and a temperature of 400 F are cooled to 150 F at constant volume. What is (a) the final pressure; (b) the quantity of heat transferred; (c) the work; (d) the change in internal energy?

4. A perfect gas is such that R = 386 and k = 1.659. If 600 Btu are added to 6 lb of the gas at constant volume when the

4. (cont'd)

initial temperature is 120 F, find (a) T_2; (b) ΔH; (c) ΔS; (d) ΔU; (e) W.

5. During a nonflow gas compression, the internal energy of a gas decreases 20 Btu while 20,000 ft-lb of work are performed on the gas. Is heat added or removed during the process? How much?

6. A reversible polytropic nonflow compression of 1 lb air proceeds from a volume of 10 ft^3 and a pressure of 50 psia to a volume of 2 ft^3. If n = 1.2 for the process, what is p2?

7. An ideal gas expands isentropically from (1) t = 539 F, to (2) p_2 = 30 psia, t_2 = 40 F. Then it is compressed reversible isothermally to (3) V_3 = 3 ft^3. The gas is then heated at constant volume to its original state. c_p = 0.30 Btu/lb F, c_v = 0.222 Btu/lb F, R = 62.2 ft-lb/lb R. (a) Draw a p - V diagram of the processes. (b) Find the work for each process and the net work for the series. (c) Determine the heat transferred for all processes and the net heat transferred for the series.

8. Two ft^3 of hydrogen at 100 psia and 300 F are expanded isentropically to 20 psia. The gas is then expanded at constant pressure, after which it is compressed by means of a reversible polytropic process with n = 1.15 to its original state. (a) Draw a p - V diagram for the series of processes. (b) Determine the net work performed. (c) Determine the change in internal energy for the series.

George Howey

Heat & Thermodynamics

VARIABLE SPECIFIC HEATS OF GASES

The heat capacities and specific heat of real gases are not constant, as was assumed to be the case of ideal gases, but vary with temperature and pressure.

Specific heat curves may be used whenever a continuous function is not required. Fig. 1, showing the variation in constant volume specific heats with pressure, indicates that pressure has little effect upon the constant volume of specific heats. Fig. 2 presents the variation of the constant pressure specific heat of air with temperature and pressure.

Example 1:

What is the amount of heat required to change the temperature of 10 lb air from 600 F to 1,000 F by constant pressure heat addition at (a) $p = 0$ psia, and (b) $p = 5{,}000$ psia?

Solution:

(a) From Fig. 2 the average specific heat at $p = 0$ from 600 F to 1,000 F is 0.257 Btu/lb F

$$Q = wc_p\ (T_2 - T_1)$$

Q = 10 lb x 0.257 Btu/lb F x (1,000 - 600) F = 1,028 Btu

(b) From Fig. 2 the average value of c_p between 600 F and 1,000 F at 5,000 psia is 0.273 Btu/lb F

Q = 10 lb x 0.273 Btu/lb F x (1,000 - 600) F = 1,092 Btu

Example 2:

How much heat is required to increase the temperature of 15 lb oxygen from 2,000 to 3,000 F by a constant volume process?

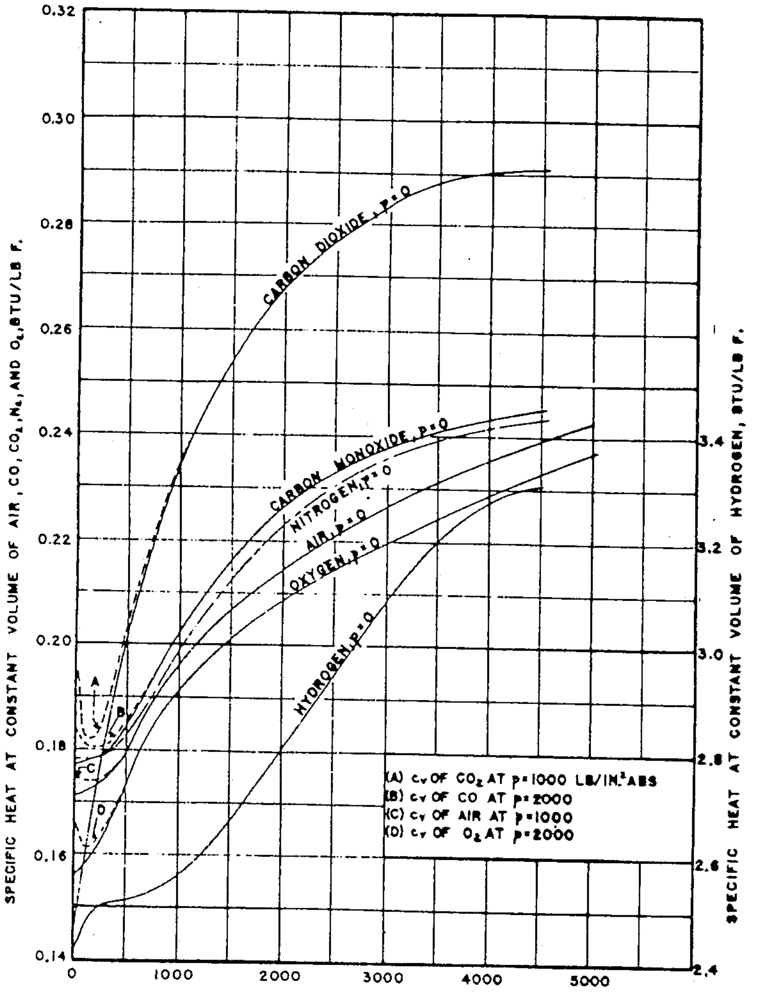
Solution:

From Fig. 1 the average c_v between 2,000 and 3,000 F is 0.214 Btu/lb F

$Q = wc_v(t_2 - t_1)$ = 15 lb x 0.214 Btu/lb F x (3,000 - 2,000) F = 3,210 Btu

Specific Heat Equations

From Figs. 1 and 2 it is evident that whenever the temperature range is large, especially at low pressure, accurate values of heat can be determined only by integration, because of the wide variation in specific heat values. The integration may be performed graphically or by integral calculus when the equations of specific heat curves are available. Several empirical molar heat capacity equations are represented in Table I. These equations give precise results at pressures near 1 atmosphere.

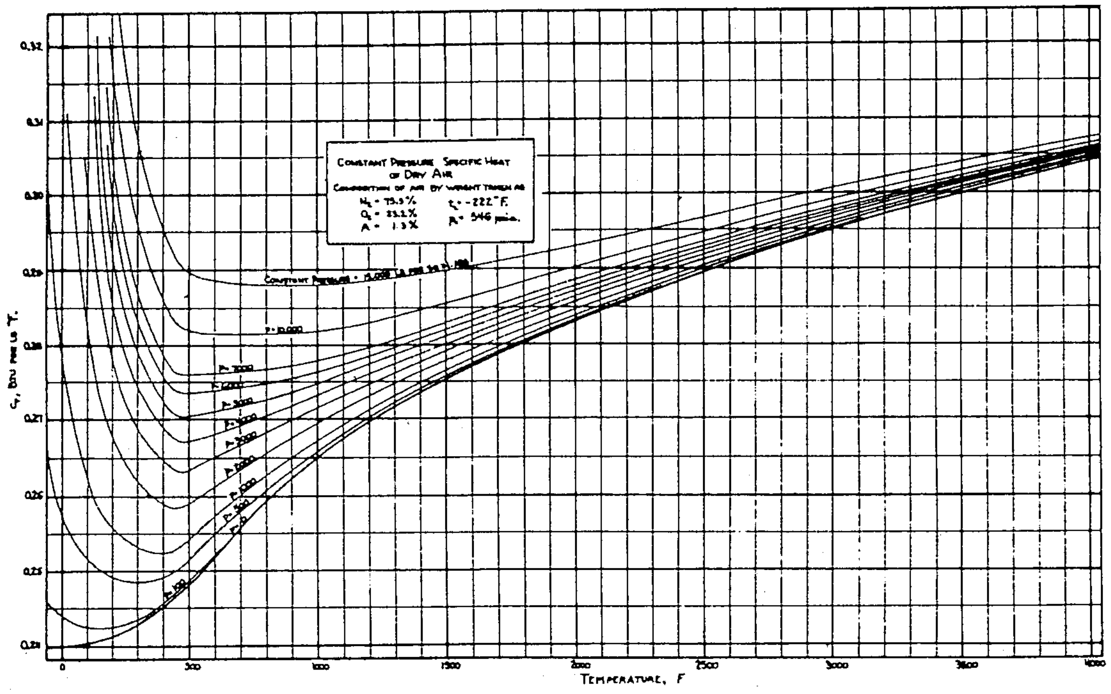


The effect of temperature and pressure on c_v various gases

Fig. 1

TABLE I

Gas	Equation (C_p in Btu/lb mole F)
O_2	$C_p = 11.515 - \frac{172}{\sqrt{T}} + \frac{1530}{T}$
N_2	$C_p = 9.47 - \frac{3.47 \times 10^3}{T} + \frac{1.16 \times 10^6}{T^2}$
H_2O	$C_p = 19.86 - \frac{579}{\sqrt{T}} + \frac{20}{T}$
CO_2	$C_p = 16.2 - \frac{6.53 \times 10^3}{T} + \frac{1.41 \times 10^6}{T^2}$



The effect of temperature on c_p of air at various pressures

Fig. 2

In chemical thermodynamic calculations, it is advisable to use one form of specific heat equations such as equation (1) for all gases

$$C_p = a + bT + cT^2 + \ldots \qquad (1)$$

When a large number of gases are involved, this form of equation proves advantageous because the equation for a mixture of gases is of the same form as the specific heat of one component. The equation for a mixture is:

$$C_p = X_A C_{PA} + X_B C_{PB} + \ldots \qquad (2)$$

where X_A, X_B, X_C etc. are the mole fractions of the components A, B, C, etc. having individual molar heat capacities C_{PA}, C_{PB}, C_{PC}, etc.

Combining equations (1) and (2) gives:

$$C_{P_{mix}} = (X_A a_A + X_B a_B + \ldots) + (X_A b_A + X_B b_B + \ldots)$$

$$T + (X_A c_A + X_B c_B + \ldots) T^2 \qquad (3)$$

TABLE II

Approximate molar heat equation constants
(C_p in Btu/lb mole R units)

	a	b x 10^3	c x 10^6
Air	6.36	0.993	-0.0852
O_2	6.100	1.810	-0.313
CO	6.342	1.020	-0.0866
H_2	6.950	-0.110	0.148

Example 3:

Determine the heat required to heat 25 lb O_2 from 500 R to 2,500 R using (a) the equation of Table I, and (b) the equation of Table II.

Solution:

$$Q = n \int_{T_1}^{T_2} C_p dT = n \int_{500}^{2500} C_p dT$$

$$n = \text{number of moles } \frac{25 \text{ lb}}{32 \text{ lb/lb mole}} = 0.782 \text{ lb mole}$$

(a) From Table I, $C_p = 11.515 - \dfrac{172}{T^{\frac{1}{2}}} + \dfrac{1530}{T}$

$$Q = 0.782 \int_{500}^{2500} \left(11.515 - \frac{172}{T^{\frac{1}{2}}} + \frac{1530}{T}\right) dT = 0.782$$

$$\left[11.515\, T - \frac{172\, T^{\frac{1}{2}}}{\frac{1}{2}} + 1530 \ln T\right]_{500}^{2500} = 12{,}550 \text{ Btu}$$

(b) From Table II, $C_p = 6.095 + 1.809 \times 10^{-3}\, T - 0.313 \times 10^{-6}\, T^2$

(b) (cont'd)

$$Q = 0.782 \text{ lb mole} \int_{500}^{2500}$$

$$(6.095 + 1.809 \times 10^{-3}T - 0.313 \times 10^{-6}T^2)$$

$$\text{Btu/lb mole RdT} = 0.782$$

$$\left[6.095T + \frac{1.809 \times 10^{-3}T^2}{2} - \frac{0.313 \times 10^{-6}T^3}{3}\right]_{500}^{2500}$$

$$Q = 12{,}550 \text{ Btu}$$

Mean Specific Heats of Gases

The specific heats obtained by the equations of Table I represent values at any temperature T. It is often advantageous when determining the total energy required to heat a gas from one temperature to a number of different temperatures to use average specific heat values over the specific temperature range involved. The total heat required can be readily determined by multiplying the mean specific heat by the temperature rise and the number of pounds of gas. The specific heat c_{av} may be obtained by equation (4).

$$c_{av} = \frac{\int_{T_1}^{T_2} cdT}{T_2 - T_1} \quad \text{or} \quad C_{av} = \frac{\int_{T_1}^{T_2} CdT}{T_2 - T_1} \qquad (4)$$

ASSIGNMENT

1. Fifty lb N_2 are heated from 40 F to 240 F at constant pressure

   (a) How much heat is required when computed by using a constant specific heat of 0.245 Btu/lb R?

   (b) How much heat is required when the variable specific heat of Table I is used?

2. Twenty lb air are heated at constant volume from 800 F to 1,600 F. What quantity of heat is required, assuming constant specific heat?

3. What is the value of k for air at 14.7 psia and 3,000 F, assuming variable specific heat?

4. What is the change in enthalpy of 1 lb CO for a change in temperature from 140 F to 300 F at 1 atm pressure? Use specific heat given in Table II.

5. Using the steam table enthalpy values, determine the constant pressure specific heat of steam at 1 atm pressure between 400 F and 460 F.

George Howey

Heat & Thermodynamics

PRESSURE, VOLUME, TEMPERATURE RELATIONSHIPS OF GASES

The pressure, volume, and temperature of gases in vapours in various states are related by equations of state assumed to be valid for the temperature and pressure range under consideration. The equations vary in their degree of complexity, requiring from 1 to 15 or more constants.

Ideal Gas Equation

The ideal equation of state, the simplest of all equations of state, may be developed from the kinetic theory of gases or from the combination of Boyles and Charles Law. It is assumed that all molecules are perfectly elastic spheres moving at constant velocity with respect to time and space, in the absence of all attractive forces between the molecules, and that the molecules occupy no space. The resulting equation is

$$pV = nR_oT \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Example 1:

Determine the pressure at 800 R of CO_2 having a molecular volume of 3.2 ft^3/lb mole.

Solution:

$$p = 1544 \text{ ft-lb/lb mole R} \times \frac{800 \text{ R}}{3.2 \text{ ft}^3\text{/lb mole}} = 386{,}000 \text{ psf}$$

$$= 183 \text{ atm}$$

one atmosphere = 2110 psf

The ideal gas equation of state may be used for the first approximation calculations when more exact information is not available.

Other Equations of State

It soon became evident to early observers that real gases deviated from the ideal equation of state. A large number of equations are in use now, that try to account for the fact that molecules do occupy space, that molecules do exert a molecular attraction upon each other. One of the earliest equations is van der Waals'

$$p = \frac{R_oT}{v - b} - \frac{a}{v^2} \quad \text{or} \quad (p + \frac{a}{v^2})(v - b) = R_oT \quad ....(2)$$

Values of a and b, the constant in van der Waals' equation, are given in Table 1.

In the absence of experimental values, approximate values given by the following equations may be used.

$$a = \frac{27}{64}\left(\frac{R_o{}^2T_c{}^2}{p_c}\right); \qquad b = \frac{R_oT_c}{8p_c}$$

where p_c and T_c represent the critical pressure and temperature, respectively.

Example 2:

Determine the pressure at 800 R of CO_2 having a molar volume of 3.2 ft^3/lb mole by using van der Waals' equation. It should be that

$$R_o = 1544 \text{ ft-lb/lb mole R} \times \frac{1}{2110 \text{ psf/atm}} = 0.730 \text{ atm ft}^3\text{/lb mole R}$$

Solution: From Table 1

$$a = 926 \text{ atm ft}^6/(\text{lb mole})^2; \qquad b = 0.686 \text{ ft}^3/\text{lb mole}$$

$$p = \frac{0.730 \text{ atm ft}^3/\text{lb mole R} \times 800\text{R}}{3.2 \text{ ft}^3/\text{lb mole} - 0.686 \text{ ft}^3/\text{lb mole}} - \frac{926 \text{ atm ft}^6/(\text{lb mole})^2}{(3.2 \text{ ft}^3/\text{lb mole})^2} =$$

$$233 \text{ atm} - 90 \text{ atm} = 143 \text{ atm}$$

Other equations of state are:

Dieterici

$$p = \frac{R_oT}{v - b}\left(e\right)^{-a/R_oTv} \quad ....(3)$$

$$a = \frac{4(R_o)^2T_c{}^2}{p_ce^2}; \qquad b = \frac{R_oT_c}{p_ce^2}$$

Bertholot

$$p = \frac{R_oT}{v - b} - \frac{a}{Tv^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$a = \frac{27}{64}\left(\frac{(R_o)^2\ T_c{}^3}{p_c}\right) \quad ; \quad b = \frac{9R_oT_c}{128p_c}$$

The Beattie, Bridgeman, and Beattie equations using the same empirical constants are available when greater accuracy than obtainable from the preceding equations is required.

Beattie, Bridgeman

$$p = R_oT\ \frac{(1 - \epsilon)}{v^2}\ (v + B) - \frac{A}{v^2} \quad \ldots\ldots\ldots\ldots (5)$$

$$A = A_o\left(1 - \frac{a}{v}\right) \quad ; \quad B = B_o\left(1 - \frac{b}{v}\right) \quad ; \quad \epsilon = \frac{c}{vT^3}$$

Beattie

$$v = (\pi = B)(1 - \epsilon) - \frac{A}{R_oT} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$A = A_o\left(1 - \frac{a}{\pi}\right) \quad ; \quad B = B_o\left(1 - \frac{b}{\pi}\right) \quad ; \quad \epsilon = \frac{c}{\pi T^3} \quad ; \quad \pi = \frac{R_oT}{p}$$

The values of the constants A_o, B_o, a, b, and c for several gases may be found in Table 1.

TABLE 1

Gas	van der Waals' constants		Beattie-Bridgeman constants					Critical constants	
	a	b	A_0	a	B_0	b	$c \times 10^{-4}$	Critical temperature	Critical pressure
	atm ft^6/(lb mole)^2	ft^3/lb mole	atm ft^6/(lb mole)^2	ft^3/lb mole	ft^3/lb mole	ft^3/lb mole	ft^{3}R^3/lb mole	R	atm
Air	344	0.587	344	0.309	0.738	−0.0176	406	238.8	37.2
Carbon dioxide	926	0.686	1285	1.14	1.69	1.16	6170	547.8	72.9
Carbon monoxide	381	0.639	345	0.419	0.808	−0.111	393	242.2	34.6
Hydrogen	62.8	0.427	50.7	−0.081	0 336	−0 698	4.7	60.5	12.8
Nitrogen	346	0.618	345	0.419	0 808	− 0 111	393	227.2	33.5
Oxygen	350	0.510	383	0.411	0 741	0 0674	449	278.1	49.7
Water vapor	1400	0.488						1165.4	218.2

Example 3:

Using the Beattie-Bridgeman equation, determine the pressure of CO_2 at 800 R when the molar volume is 3.2 ft^3/lb mole.

Solution:

From Table 1 the critical temperature and pressure are 547.8 R and 72.9 atm, respectively. The constants for the Beattie-Bridgeman equation are obtained from Table 1.

$A_o = 1285$; $a = 1.14$; $B_o = 1.69$; $b = 1.16$; $c = 6170 \times 10^{-4}$

$$A = 1285 \left(1 - \frac{1.14}{3.2}\right) = 826; \quad B = 1.69 \left(1 - \frac{1.16}{3.2}\right) = 1.08$$

$$\epsilon = \frac{6170 \times 10^{-4}}{3.2 \times (800)^3} = 0$$

Using equation 5 to calculate the pressure gives:

$$p = 0.73 \times 800\left(\frac{1}{3.2}\right)^2 \times (3.2 + 1.08) - \frac{826}{(3.2)^2} = 244 - 81$$
$$= 163 \text{ atm}$$

Compressibility Factors and Reduced Coordinates

The ideal gas equation may be modified by a compressibility factor for use with real gases:

$$pv = ZR_oT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Unfortunately no simple relationship exists between the comressibility factor, pressure, and temperature; this necessitates determination of a compressibility factor for each state of the gas. It is customary to present compressibility factor data by using plots of Z as a function of pressure for various constant temperatures. By employing the law of corresponding states (gases at the same reduced coordinates have approximately the same properties), it is possible to present the compressibility data for all gases on one diagram.

The reduced properties may be obtained by using equations (8) and (10).

$$\text{Reduced pressure } p_r = \frac{\text{actual pressure}}{\text{critical pressure}} = \frac{p}{p_c} \quad \ldots.(8)$$

$$\text{Reduced temperature } T_r = \frac{\text{actual temperature}}{\text{critical temperature}} = \frac{T}{T_c} \quad .(9)$$

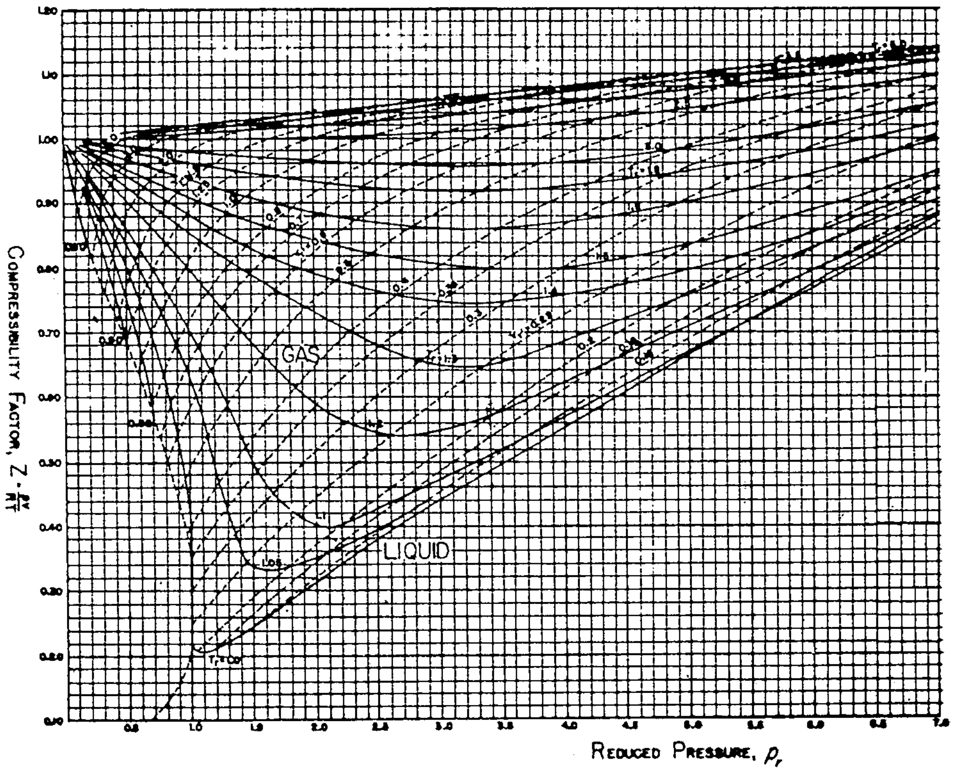
$$\text{Reduced volume } v_r = \frac{\text{actual volume}}{\text{critical volume}} = \frac{v}{v_c} \quad ..(10)$$

Because of the lack of sufficient critical volume data, an idealized reduced volume v_{ri} defined by equation (11) is sometimes employed.

$$v_{ri} = \frac{v}{RT_c/p_c} \quad ....(11)$$

where v = actual volume.

Fig. 1 is a generalized compressibility chart which includes v_{ri} values as well as the customary T_r, p_r, and Z data.



Compressibility Chart

Fig. 1

Example 4:

Using the compressibility factor, determine the pressure at 800 R of CO_2 having a molar volume of 3.2 ft^3/lb mole.

Solution:

From Table 1, $T_c = 547.8$ R, and $p_c = 72.9$ atm. Calculate the reduced temperature and ideal reduced volume, using equations (9) and (11) respectively.

$$T_r = \frac{800}{547.8} = 1.46; \qquad v_{ri} = \frac{3.2}{(0.73 \times 547.8)/72.9} = 0.585$$

From Fig. 1, Z = 0.82. By equation (9):

$$p = \frac{0.82 \times 0.73 \times 800}{3.2} = 148 \text{ atm}$$

Check $p_r = \frac{148}{72.9} = 2.08$; from Fig. 1, Z = 0.82. Solution satisfactory.

Generalized Equation of State

Several equations of state using reduced coordinates have been proposed. One of the latest is the Su equation.

$$p_r = \frac{T_r}{v_{ri} - 0.125} - \frac{0.42}{(v_{ri})^2} \quad \text{.................(12)}$$

Example 5:

Using equation (12) determine the pressure at 800 R of CO_2 having a molar volume of 3.2 ft^3/lb mole.

Solution:

From equation (9)

$$T_r = \frac{800}{547.8} = 1.46$$

From equation (11)

$$v_{ri} = \frac{3.2}{(0.73 \times 547.8)/72.9} = 0.585$$

Using equation (12):

$$p_r = \frac{1.46}{0.585 - 0.125} - \frac{0.42}{(0.585)^2} = 3.17 - 1.23 = 1.94$$

From equation (8):

$$p = 1.94 \text{ x } 72.8 \text{ atm} = 141 \text{ atm}$$

Real Gas Mixtures

Only three of the many available methods of treating real gas mixtures will be outlined briefly in the following: They are: (1) the law of additive pressures, (2) the law of additive volumes, and (3) the pseudocritical temperature and pressure method.

METHOD 1:

The law of additive pressures assumes that the pressure of a gas mixture is the sum of the pressures exerted by the individual components if each is assumed to occupy the total volume of the mixture alone. The components may be assumed to follow any one of the many equations of state. For example, if a mixture of x_A moles of a gas A, x_B moles of gas B, etc., in total volume V is assumed to follow the van der Waals' equation, the following equation results:

$$p_A = \frac{x_A R_o T}{V - x_A b_A} - \frac{a_A x_A^2}{V^2} \text{ ; } p_B = \frac{x_B R_o T}{V - x_B b_B} - \frac{a_B x_B^2}{V^2}\text{; etc } ..(13)$$

Total pressure of mixture:

$$p = p_A + p_B + p_C + \ldots.$$

METHOD 2:

The law of additive volumes assumes that each component of a gas mixture is at the temperature and pressure of the mixture, and that the total volume of the mixture is equal to the sum of the individual component volumes. This method leads to a trial-and-error solution. Here again any equation of state may be employed. Assume that v is the molar volume of the gas mixture, and that v_A and v_B are the molar volumes of the components A and B at the temperature and pressure of the mixture. Let N_A and N_B represent the mole fractions of the components. Then by the law of additive volumes:

$$v = N_A v_A + N_B v_B \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Assume a value for v_A or v_B and solve equation (14) for the remaining molar volume, then substitute these values in the chosen equation of state and solve for the pressure of the components. When the component pressures are equal, a solution has been obtained.

METHOD 3:

The pseudocritical temperature and pressure method treats a gas mixture as though it were a single gas having a critical temperature and pressure equal to the pseudocritical temperature and pressure of the mixture. This pseudocritical point does not in general coincide with the true critical point of the mixture. The compressibility factor for the mixture is obtained from the generalized reduced coordinate compressibility factor chart, Fig. 1, using the pseudocritical temperature and pressure in computing the reduced temperature and pressure of the mixture. Equation (7) is then used for the gas mixture.

Equation (15) and (16) may be used to compute the pseudocritical values. More complicated equations may be found in reference 8.

$$p_c' = N_A p_{c_A} + N_B p_{c_B} + N_C p_{c_C} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$T_c' = N_A T_{c_A} + N_B T_{c_B} + N_C T_{c_C} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

where

p_{c_A} = critical pressure of component A, etc

T_{c_A} = critical temperature of component A, etc.

N_A = mole fraction of component A, etc.

Example 6:

Determine the pressure of a mixture of 0.7997 mole fraction of methane and 0.2003 mole fraction of n - butane at 340 F, the mixture having a molar volume of 4.511 ft^3/lb mole.

Solution:

(a) $$p = \frac{R_oT}{v} = \frac{0.73 \text{ atm ft}^3/\text{lb mole R} \times 800 \text{ R}}{4.511 \text{ ft}^3/\text{lb mole R}} = 129.5 \text{ atm,}$$

or 1900 psia

(b) Law of additive pressure and van der Waals' equation. Using equation (13) gives:

$$p_{CH_4} = \frac{0.73 \times 800 \times 0.7997}{4.511 - 0.6855\ (0.7997)} - \frac{581.2 \times (0.7997)^2}{(4.511)^2}$$

$$= 99.5 \text{ atm}$$

$$pC_4H_{10} = \frac{0.73 \times 800 \times 0.2009}{4.511 - 1.944\ (0.2009)} - \frac{3675\ (0.2009)^2}{(4.511)^2}$$

$$= 21.2 \text{ atm}$$

p_T = 99.5 + 21.2 = 120.7 atm, or 1770 psia

(c) Law of additive volumes and van der Waals' equation. By equation (14):

$$4.511 = 0.7997\ v_{CH_4} + 0.2003\ vC_4H_{10}$$

After several trials, assume vCH_4 = 4.85 ft^3/lb mole. Then by the above equation, vC_4H_{10} = 3.15 ft^3/lb mole. Using van der Waals' equation gives:

$$p_{CH_4} = \frac{0.73 \times 600}{4.85 - 0.68} - \frac{581.2}{(4.85)^2} = 114 \text{ atm}$$

$$pC_4H_{10} = \frac{0.73 \times 800}{3.15 - 1.94} - \frac{3675}{(3.15)^2} = 112 \text{ atm}$$

These values are in good agreement; therefore the assumed molar volumes are correct, and the pressure of the mixture is 113 atm, or 1660 psia.

(d) Pseudocritical temperature and pressure. By equations (15) and (16):

$$T'_c = 0.7997 \times 380 + 0.2003 \times 770 = 458 \text{ R}$$

$$p'_c = 0.7997 \times 45.8 + 0.2003 \times 36 = 43.9 \text{ atm}$$

$$T'_r = \frac{T}{T'_c} = \frac{800}{458} = 1.75$$

$$v'_{ri} = \frac{v}{\frac{R_o T'_c}{p'_c}} = \frac{4.511}{\frac{0.73 \times 458}{43.9}} = 0.592$$

from Fig. 1, Z = 0.91. By equation (7),

$$p = \frac{0.91 \times 0.73 \times 800}{4.511} = 117.5 \text{ atm, or } 1730 \text{ psia}$$

ASSIGNMENT

1. Determine the pressure of 1 lb mole of nitrogen at 910 R when the molar volume is 1.40 ft^3/lb mole, by means of the ideal gas equation.
2. Repeat problem 1, using the compressibility chart.
3. Repeat problem 1, using the Bertholot equation.
4. Repeat problem 1, using the Beattie, Bridgeman equation.
5. Repeat problem 1, using the Su equation.
6. Determine the molar volume at 313 K of CO_2 having a pressure of 72.16 atm using (a) van der Waals' equation; (b) ideal gas equation; (c) Beattie equation.

George Howey

Heat & Thermodynamics

THE SECOND LAW OF THERMODYNAMICS AND ENTROPY

The first law of thermodynamics is used to relate and to evaluate the quantities of the various types of energy involved in a process. However, no information as to the direction of the process can be obtained by application of the first law. Early in the development of the science of thermodynamics, investigators noted that while work could be converted completely into heat, the converse was never true for a cyclic process. Certain natural processes were also observed to proceed always in a certain direction. The second law of thermodynamics was developed as an explanation for these natural phenomena.

One consequence of the second law is the development of a physical property of matter termed entropy. The change in this property, which may be defined as the index of unavailable energy, can be used to determine the direction in which a given process will proceed. A discussion of the second law and the development of a concept of entropy will be presented in this lesson.

Statements of the Second Law

Several common statements of the second law are:

1. It is impossible to construct a machine that, while operating in a cycle, will produce no other effect than the transfer of heat from a cold body to a body at a higher temperature.
2. It is impossible to construct an engine that, while operating in a cycle, will absorb heat from a single reservoir and produce an equivalent amount of work.
3. A simplified statement of the second law may be presented as: heat by itself will not flow from a cold to a hot body.

Reversible and Irreversible Processes

Thermodynamic problems, processes, and cycles are often advantageously investigated by substitution of reversible processes for the actual irreversible processes. As has been pointed out before, changes in physical properties are independent of the path; therefore a series of reversible processes may be substituted for the actual irreversible process and thus aid the engineer in his study. Several statements defining reversible and irreversible processes are given in the following:

"In order for a process to be reversible, the following conditions must be completely and precisely fulfilled.

1. The process may proceed in either direction.
2. The process may be reversed by infinitely small changes in the values of the conditions controlling it.
3. The working substance is always in a state of equilibrium throughout the process.
4. No energy transformations occur as a result of friction.
5. All energy transformations which occur as the process proceeds in one direction must equal in both form and amount those which take place in the reverse order.
6. The working substance must be returned to the initial condition by retracing every step of the original process in the reverse order."

"A process is irreversible if a perpetual motion machine of the second kind would result from its being reversible."

From these statements it follows that only reversible processes can be depicted on the p-v, T-s, etc, diagrams. Actual or irreversible processes cannot be drawn, since they are not a succession of equilibrium conditions. Only the initial and final conditions of irreversible processes are known; however, some authors prefer to represent the irreversible process by dotted lines on the diagrams.

Example 1:

Classify the following processes as reversible or irreversible and state the reason for the classification. (a) Free expansion of a gas. (b) The transfer of heat from a body at a temperature T + dT to a body at temperature T.

Solution:

(a) A gas is said to undergo free expansion when it expands from a state of p_1V_1 at constant temperature to a state p_2V_2, doing no work and in the absence of heat transfer.

This process is irreversible. It can be shown that its being reversible would result in a perpetual motion machine of the second kind; therefore, it is irreversible by Keenan's irreversible process definition.

(b) This process may be proven to be reversible by means of the preceding reversible process statement. If the temperature were changed an indefinitely small amount, dT, the direction of heat transfer would be reversed.

Carnot Cycle and Principle

Sadi Carnot, in 1824, proposed a heat power cycle composed of the following reversible processes:

(1) isothermal expansion and reception of heat at temperature θ_1;

(2) isentropic expansion;

(3) isothermal compression and rejection of heat at temperature θ_2; and

(4) isentropic compression of temperature θ_1.

The Carnot cycle operating between two fixed temperatures θ_1 and θ_2 and employing gas as the working fluid is plotted on the T - s and p - v diagrams shown in Fig. 1. The efficiency of the cycle is given by the following equations:

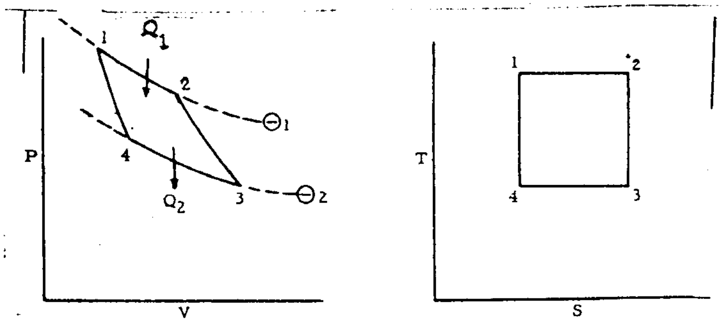


Fig. 1

$$\eta = \frac{Q_1 - Q_2}{Q_1} \qquad \text{.........................(1)}$$

where Q_1 is the heat received at temperature θ_1, and Q_2 is the heat rejected at temperature θ_2.

Since all component processes of the Carnot cycle are reversible, the complete cycle may be reversed to produce a Carnot heat pump or refrigeration cycle. This cycle absorbs heat at the low temperature θ_2, receives work input, and rejects heat at the higher temperature θ_1.

In the field of refrigeration, the term coefficient of performance is used as the criterion of performance similar to the use of thermal efficiency in the heat power field. The CP or coefficient of performance is defined as the ratio of the refrigerating effect (heat absorbed at the low temperature) to the work input. The CP for the reversed Carnot cycle is given by the following equation:

$$CP = \frac{Q_2}{Q_1 - Q_2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

where Q_2 is the heat absorbed by the reversed Carnot cycle at temperature θ_2, which is equivalent to the heat rejected by the Carnot engine, and Q_1 is the heat rejected by the reversed Carnot engine at temperature θ_1, which is equivalent to the heat absorbed by the Carnot engine.

Example 2:

A Carnot engine receives 100 Btu and rejects 75. What is the efficiency?

Solution:

Using equation (1) gives:

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{100 \text{ Btu} - 75 \text{ Btu}}{100 \text{ Btu}} \times 100 = \frac{25 \text{ Btu}}{100 \text{ Btu}} \times 100 = 25\%$$

Example 3:

The Carnot engine of example 2 is reversed to form a Carnot refrigerator, with 75 Btu being absorbed at the low temperature and 100 Btu being rejected. Determine the coefficient of performance.

Solution: Using equation (2) gives:

$$CP = \frac{Q_2}{Q_1 - Q_2} = \frac{75 \text{ Btu}}{100 \text{ Btu} - 75 \text{ Btu}} = \frac{75 \text{ Btu}}{25 \text{ Btu}} = 3$$

Carnot also disclosed the following principle which bears his name today:

1. No engine can be more efficient than a reversible engine operating between the same high-temperature and low-temperature reservoirs. Here the term heat reservoir is taken to mean either a heat source or a heat receiver or sink.

2. The efficiencies of all reversible engines operating between the same constant temperature reservoirs are the same.

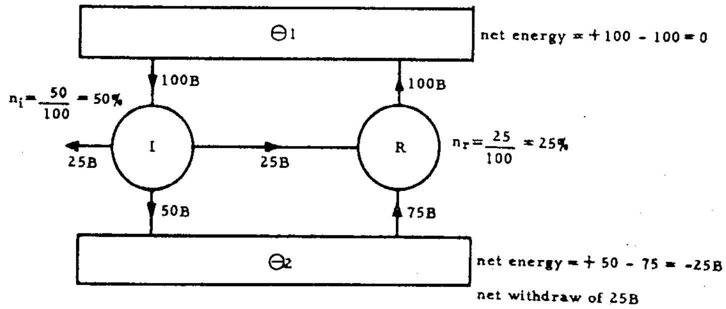
3. The efficiency of a reversible engine depends only upon the temperatures of the heat source and heat receiver.

Example 4:

Prove that the first statement of Carnot's principle is true.

Solution:

Assume that the statement is false and that an irreversible engine can be more efficient than a reversible engine operating between temperatures θ_1 and θ_2. Assume further that both engines receive 100 Btu and that the irreversible engine rejects 50 Btu, whereas the reversible engine rejects 75 Btu. Reverse the reversible engine to act as a refrigerator and drive it by the irreversible engine. The reversed engine will now receive 25 Btu work input, will absorb 75 Btu from the low-temperature heat reservoir and will reject 100 Btu to the high-temperature reservoir. The irreversible engine will absorb 100 Btu from the high-temperature reservoir deliver 50 Btu of work output, 25 Btu of which are used to drive the reversed reversible engine, and 25 Btu constitute net useful output; and reject 50 Btu to the low-temperature reservoir. The net result of the operation of the combination of the irreversible and reversible engines is the absorption of 25 Btu from the low-temperature reservoir and the production of 25 Btu of useful work. This is a perpetual motion machine of the second kind and therefore impossible. Carnot's principle must be true since the negative of it cannot be true. Fig. 2 illustrates the combined irreversible and reversible engines.



Irreversible and reversible engines operating between same heat source and sink

Fig. 2

Kelvin Temperature Scale

In 1848 Lord Kelvin derived a temperature scale independent of the nature of the thermometric substance, by basing it upon the Carnot engine. He defined the scale such that:

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where Q_1 is the heat received by a Carnot engine from a source at temperature T_1, and Q_2 is the heat rejected to a receiver at temperature T_2.

The combination of equations (3) and (1) results in an expression for the Carnot engine efficiency in terms of the temperatures of the heat reservoirs:

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The magnitude of the unit of temperature is determined by the number of Carnot engines interposed between two given reference temperatures. One advantage of the absolute scale is the absence of negative temperatures.

Example 5:

Determine the efficiency of a Carnot engine operating between the following temperatures: (a) 1040°F and 40°F, (b) 1040°F and -60°F, and (c) 1140°F and 40°F.

Solution:

The following results are obtained using equation (4):

(a) $T_1 = 1040^\circ F + 460 = 1500^\circ R$, $T_2 = 40^\circ F + 460 = 500^\circ R$

$$\eta = \frac{1500 - 500}{1500} \times 100\% = \frac{1000}{1500} \times 100\% = 66\%$$

(b) $T_1 = 1040^\circ F + 460 = 1500^\circ R$; $T_2 = -60 + 460 = 400^\circ R$

$$\eta = \frac{1500 - 400}{1500} \times 100\% = \frac{1100}{1500} \times 100\% = 73.5\%$$

(c) $T_1 = 1140^\circ F + 460 = 1600^\circ R$; $T_2 = 40^\circ F + 460 = 500^\circ R$

$$\eta = \frac{1600 - 500}{1600} \times 100\% = \frac{1100}{1600} \times 100\% = 68.9\%$$

Note that the temperatures must be expressed in degrees absolute, and that the reduction of the receiver temperature is more effective in increasing the efficiency than raising the source temperature an equivalent amount.

Example 6:

An inventor claims to have an engine which receives 100 Btu and produces 25 Btu of useful work when operating between a source at 140°F and a receiver at 0°F. Is this claim valid?

Solution:

The maximum efficiency obtainable by any engine operating between the given temperatures is that of the reversible or Carnot engine. Therefore, the maximum efficiency of the inventor's engine, if it were reversible, is given by equation (4):

$$T_1 = 140^\circ F + 460 = 600^\circ R; \quad T_2 = 0^\circ F + 460 = 460^\circ R$$

$$\eta = \frac{600^\circ R - 460^\circ R}{600^\circ R} \times 100\% = \frac{140^\circ R}{600^\circ R} \times 100\% = 23.3\%$$

The maximum efficiency is less than the claimed efficiency; therefore the claim is invalid.

Example 7:

Determine the coefficient of performance of a Carnot refrigerator operating between 0°F and 80°F.

Solution:

Expressing the temperature in degrees Kelvin gives:

$$T_1 = 80^\circ C + 273 = 353^\circ K \text{ and } T_2 = 0^\circ C + 273 = 273^\circ K$$

Using equations (3) and (2) gives:

$$CP = \frac{T_2}{T_1 - T_2} = \frac{273}{353 - 273} = \frac{273}{80} = 3.41$$

Clausius' Inequality and Entropy

Clausius noted that equation (3) could be rearranged as follows:

$$\frac{Q_1}{T_1} = \frac{-Q_2}{T_2} \quad \text{or} \quad \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The minus sign of Q_2 denotes heat leaving the system. This equation indicates that the change in the quantity Q/T around a reversible or Carnot cycle is zero. Clausius extended this relationship to embrace all cyclic processes by stating that the cyclic integral of $\delta Q/T$ is less than zero or in the limit equal to zero; thus:

$$\oint \frac{\delta Q}{T} \lesseqgtr 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

This equation is now known as the Inequality of Clausius.

It can be proven that the cyclic integral of $\delta Q/T$ is equal to zero for all reversible cycles and less than zero for all irreversible cycles. Since the summation of the quantity $\delta Q/T$ for a reversible cycle is equal to zero, it follows that the value of the integral of $\delta Q/T$ <u>is the same for any reversible process between states 1 and 2 of a system, and thus is a property of the system</u>.

This property of a system is called entropy and may be defined as the ratio of heat transferred during a reversible process to the absolute temperature of the system, or expressing it mathematically:

$$ds = \left(\frac{\delta Q}{T}\right)_{rev}, \quad \text{or} \quad \Delta s = \int_{rev} \frac{\delta Q}{T} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Entropy, since it is a property, is advantageously used as one of the coordinates when representing a <u>reversible</u> process graphically. Examples of its uses may be found in the T-s and h-s diagrams already discussed. Equation (7) indicates that Tds = δQ; therefore the area under <u>a reversible process curve</u> on the T-s diagram represents the quantity of heat transferred during the process. Some authors maintain that only reversible processes can be plotted; therefore they indicate irreversible processes by dotted lines. The reader should note that the area under an indicated irreversible process on a T-s diagram does not represent a quantity of heat.

The change in entropy may be positive or negative depending upon the direction of energy exchange. If 10 Btu of heat leaves a system at 500°R or is received by the surroundings at 500°R, the entropy change for the system, by equation (7), is:

$$\Delta S \text{ system} = \frac{-10 \text{ Btu}}{500^{\circ}\text{R}} = -0.20 \text{ Btu}/^{\circ}\text{R}$$

The corresponding change in entropy of the surroundings is:

$$\Delta S \text{ surroundings} = \frac{+10 \text{ Btu}}{500^{\circ}\text{R}} + 0.20 \text{ Btu}/^{\circ}\text{R}$$

The algebraic sum of the entropy change of the system and surroundings may be termed the entropy change of the universe; thus:

$$\Delta S \text{ universe} = \Delta S \text{ system} + \Delta S \text{ surroundings} \quad \ldots\ldots\ldots\ldots (8)$$

The entropy change of the universe for the above example is:

$$\Delta S_{\text{universe}} = (-0.20) + (+0.20) = 0$$

Clausius termed the universe an isolated system, and based upon his inequality it follows that:

$$\Delta S_{\text{universe}} \geq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

This equation is also a criterion of reversibility, for it can be shown that if ΔS of the universe is equal to zero, the system has undergone a reversible process, and if ΔS of the universe is greater than zero, the system has undergone an irreversible process.

Changes in Entropy for Several Processes

The change in entropy of a system during a reversible process is calculated by means of equation (7). When the system undergoes an irreversible process between states 1 and 2, the change in entropy is obtained by summing the changes in entropy for a number of reversible processes which, when substituted for the original irreversible process, cause the system to change from state 1 to state 2. This procedure may be followed because entropy is a point function; therefore the change in entropy is independent of the path or process. The following examples are illustratrative applications of equation (7).

Example 8:

Heating. Compute the change in entropy of 1 lb water when it is heated from 32 to 212°F.

Solution:

For this process $\delta Q = wc_p dT$; $w = 1$ lb; $c_p = 1$ Btu/lb°R.

Applying equation (7) gives:

$$\Delta S = \int_{\text{Rev}} \frac{\delta Q}{T} = \int_{492}^{672} (1 \text{ lb})(1 \text{ Btu/lb}^\circ\text{R}) \frac{dT}{T} = (1 \text{ lb})(1 \text{ Btu/lb}^\circ\text{R}) \ln \frac{672}{492} =$$

$$0.311 \text{ Btu/}^\circ\text{R}$$

Example 9:

A system initially at 500°R increases in temperature by absorption of 100 Btu of heat from the surroundings initially at 1500°R. The heat capacities of the system and surroundings are 0.75 Btu/°R and 0.50 Btu/°R, respectively. Was the process reversible?

Solution:

Assume that 100 Btu of heat are added reversibly to the system. The final temperature of the system is determined as follows:

$$Q = c(T_2 - T_1), \quad \text{or} \quad T_2 = T_1 + \frac{Q}{c}$$

$$T_2 = 500 + \frac{100}{0.75} = 633°R$$

Applying equation (7) gives:

$$\Delta S_{\text{system}} = \int_{500}^{633} \frac{\delta Q}{T} = \int_{500}^{633} 0.75 \text{ Btu/°R } \frac{dT}{T} = 0.75 \text{ Btu/°R} \ln \frac{633}{500} =$$

$$+0.176 \text{ Btu/°R}$$

Assume that the surroundings loses 100 Btu of heat reversibly; then

$$-Q = c(T_2 - T_1), \quad \text{or} \quad T_2 = T_1 - \frac{Q}{c} = 1500 - \frac{100}{0.5} = 1300°R$$

The minus sign indicates that heat left the surroundings. Applying equation (7) gives:

$$\Delta S_{\text{surroundings}} = \int_{1500}^{1300} \frac{c\,dT}{T} = 0.5 \text{ Btu/°R} \ln \frac{1300}{1500} = -0.072 \text{ Btu/°R}$$

By equation (8):

$$\Delta S_{\text{universe}} + 0.176 \text{ Btu/°R} - 0.072 \text{ Btu/°R} = +0.104 \text{ Btu/°R}$$

Therefore by equation (9) the process is irreversible.

Example 10:

Friction. Compute the change in entropy of 1 lb water when its temperature changes from 32°F to 212°F during a mechanical stirring process.

Solution:

Assume that the reversible addition of a quantity of heat δQ_F causes the water to change from 32°F to 212°F. Applying equation (7) gives:

$$\Delta S = \int_{Rev} \frac{\delta Q}{T} = \int_{Rev} \frac{\delta Q_F}{T} = \int_{492}^{672} 1 \text{ lb} \times 1 \text{ Btu/lb}^{\circ}\text{R} \frac{dT}{T} = 0.311 \text{ Btu/}^{\circ}\text{R}$$

Example 11:

Mixing of liquids accompanied by a change in temperature. Three lb of liquid A at 60°F are mixed with 5 lb of liquid B at 250°F under constant pressure. Determine the change in entropy for the system composed of the two liquids, assuming that the constant pressure specific heats are 0.95 Btu/lb°R for liquid A and 0.55 Btu/lb°R for liquid B.

Solution:

The change in entropy during mixing may be obtained by considering the process to be similar to the reversible heating of liquid A and the reversible cooling of liquid B, such that the final temperature of both liquid A and liquid B is equal to the final temperature of the mixture.

The final mixture temperature T_m is obtained by equating the heat given up by liquid B to that absorbed by liquid A; thus:

$$-Q_B = Q_A \quad \text{or} \quad -c_{p_B} w_B (T_m - T_B) = c_{p_A} w_A (T_m - T_A)$$

where T_A = initial temperature of liquid A
T_B = initial temperature of liquid B
w_A = weight of liquid A
w_B = weight of liquid B

The minus sign indicates heat leaving liquid B. Substituting numerical values and solving for T_m gives:

$$\frac{-0.55\ \text{Btu}}{\text{lb}\ ^{\circ}\text{R}} \times 5\ \text{lb}(T_m - 710)^{\circ}\text{R} = \frac{0.95\ \text{Btu}}{\text{lb}\ ^{\circ}\text{R}} \times \text{lb}(T_m - 520)^{\circ}\text{R}$$

$$T_m = 613^{\circ}\text{R}$$

The entropy change of liquid A, by equation (7), is:

$$\Delta S_A = w_A \int_{T_A}^{T_m} c_{p_A} \frac{dT}{T} = 3\ \text{lb} \int_{520}^{613} (0.95\ \text{Btu/lb}^{\circ}\text{R})\ \frac{dT}{T} = (2.85\ \text{Btu}/^{\circ}\text{R})$$

$$\times \ln \frac{613}{520} = +0.472\ \text{Btu}/^{\circ}\text{R}$$

The entropy change of liquid B is:

$$\Delta S_B = w_B \int_{T_B}^{T_m} c_{p_B} \frac{dT}{T} = 5\ \text{lb} \int_{710}^{613} (0.55\ \text{Btu/lb}^{\circ}\text{R})\ \frac{dT}{T} = (2.75\ \text{Btu}/^{\circ}\text{R})$$

$$\times \ln \frac{613}{710} = -0.408\ \text{Btu}/^{\circ}\text{R}$$

$$\Delta S\ \text{Total} = \Delta SA + \Delta SB = +0.472\ \text{Btu}/^{\circ}\text{R} + (-0.408)\text{Btu}/^{\circ}\text{R}$$

$$= +0.064\ \text{Btu}/^{\circ}\text{R}$$

This example indicates that the entropy of a system increases during a mixing process.

Example 12:

Isothermal mixing of different perfect gases. Two lb moles of nitrogen at 15 psia and 70°F are mixed with 3 lb moles of oxygen at the same temperature and pressure. Determine the change in entropy for the system consisting of the two gases.

Solution:

The mixing process of the two gases may be treated as though each gas were permitted to expand reversibly and isothermally from its original volume to the total volume occupied by both gases after mixing, in the absence of any external work. The change in entropy for the system will be equal to the sum of the changes in entropy for each gas during the assumed isothermal expansion, as calculated by means of equation (21) or equation (22), lesson 4. The original volumes of the oxygen and nitrogen and the final volume of the mixture are obtained by using the ideal gas equation, lesson 4.

$$V_{N_2} = \frac{nR_oT}{p} = \frac{2 \text{ lb moles x } 1544 \text{ ft-lb/lb mole } ^oR \text{ x } 530^oR}{15 \text{ x } 144 \text{ lb/ft}^2} = 756 \text{ ft}^3$$

$$V_{o_2} = \frac{nR_oT}{p} = \frac{3 \text{ lb moles x } 1544 \text{ ft-lb/lb mole } ^oR \text{ x } 530^oR}{15 \text{ x } 144 \text{ lb/ft}^2} = 1134 \text{ ft}^3$$

$$V_{mip} = \frac{nR_oT}{p} = \frac{5 \text{ lb moles x } 1544 \text{ ft-lb/lb mole } ^oR \text{ x } 530^oR}{15 \text{ x } 144 \text{ lb/ft}^2} = 1890 \text{ ft}^3$$

Applying equation (21), lesson 4, written for w pounds gives:

$$S_2 - S_1 = \Delta S_{N_2} = w \int_{530}^{530} c_v \frac{dT}{T} + \frac{wR}{J} \int_{756}^{1890} \frac{dV}{V} = 0 + (2 \text{ lb moles})$$

$$(28 \text{ lb/lb mole}) \frac{54.99 \text{ ft-lb/lb}^oR}{778 \text{ ft-lb/Btu}} \log \frac{1890}{756} = +3.63 \text{ Btu/}^oR$$

$$\Delta S_{o_2} = w \int_{530}^{530} c_v \frac{dT}{T} + \frac{wR}{J} \int_{1134}^{1890} \frac{dV}{V} = 0 + (3 \text{ lb moles})(32 \text{ lb/lb mole})$$

$$\frac{48.25 \text{ ft-lb/lb}^oR}{778 \text{ ft-lb/Btu}} \log \frac{1890}{1134} = + 3.05 \text{ Btu/}^oR$$

$$\Delta S_{system} = \Delta S_{N_2} + \Delta S_{o_2} = 3.63 + 3.05 = 6.68 \text{ Btu/}^oR$$

Example 13:

During a reversible adiabatic or isentropic compression process 1 lb air is compressed from 540°R and 1 atm pressure to a pressure of 10 atm. Plot the process upon T-s and h-s diagrams. Plot an irreversible adiabatic compression between the same pressures upon the diagrams.

Solution:

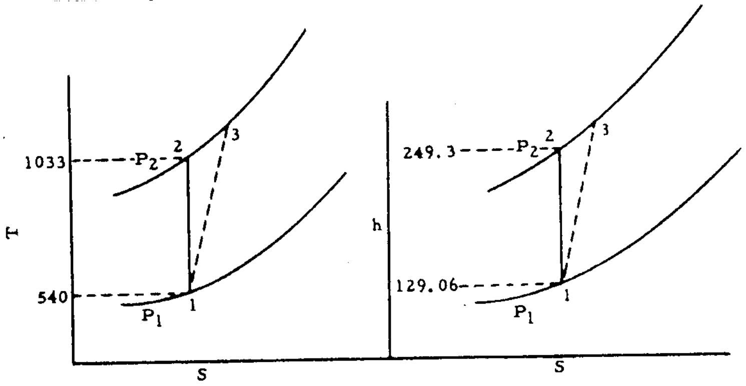
From the Keenan Gas Tables at 540°R

$$p_{r1} = 1.3860; \quad h_1 = 129.06$$

$$p_{r2} = p_{r1} \times \frac{p_2}{p_1} = 1.3860 \times \frac{10}{1} = 13.860$$

From the gas tables at $p_r = 13.860$, $T_2 = 1033$: $h_2 = 249.3$: for the isentropic process $\Delta s = 0$.

During an irreversible adiabatic compression process the entropy must increase by equation (9). Therefore the final state for the irreversible compression process will lie to the right of the state reached by the reversible compression and is represented by the dotted line on the diagram.



Air compression process plotted upon the T-S and h-S diagrams

Fig. 3

Entropy and Unavailable Energy

From the discussion of the second law it was learned that a portion of the heat supplied to a reversible engine operating in a cycle must be rejected; thus part of the energy supplied is unavailable for conversion into useful work by the most perfect engine imaginable. That portion of the heat supplied converted into useful work is termed available energy. Thus the terms available and unavailable energy refer to the conversion of heat into useful work by a perfect engine. Applying the first law to an engine gives:

$$Q_s = Q_a + Q_u \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (10)$$

where Q_s is the heat supplied, Q_a is the available energy converted into useful work, and Q_u is the unavailable energy or the energy rejected by the reversible engine.

From equation (3) it follows that:

$$\frac{Q_u}{T_u} = \frac{Q_s}{T_s} \quad \text{or} \quad Q_u = T_u \frac{Q_s}{T_s} \quad \dots\dots\dots\dots\dots\dots (11)$$

where T_u = temperature at which heat is rejected, or reference temperature

T_s = temperature at which heat is supplied

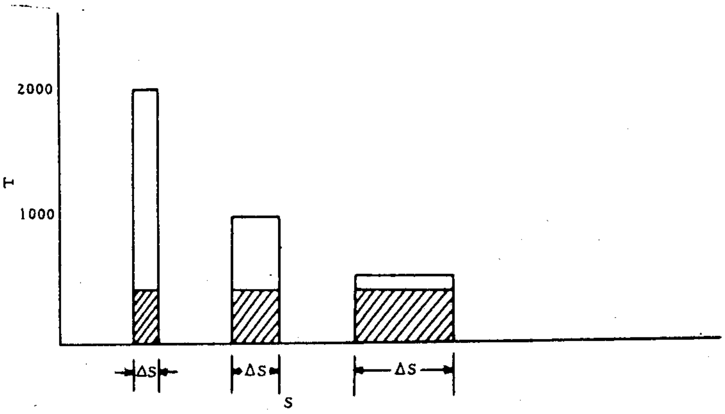
Writing equation (11) in incremental form gives:

$$\Delta Q_u = T_u \frac{\Delta Q_s}{T_s} \quad \dots\dots\dots\dots\dots\dots\dots\dots (12)$$

But $\delta Q_s/T_s$ is the change in entropy of the system during heat reception; therefore equation (12) may be written:

$$\Delta Q_u = T_u \Delta S \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (13)$$

Although equation (13) was derived for the Carnot engine, it is much more general in nature and may be applied to any process. Stated in words, equation (13) states that the increase in unavailable energy is equal to the reference or receiver temperature multiplied by the change in entropy for the process. This gives rise to the definition of entropy as the index of unavailable energy, because it can be seen from equation (13) that the unavailable energy increased directly with the change in entropy and the reference temperature.



The increase in unavailable energy as the temperature of energy reception is decreased.

Fig. 4

Example 14:

Determine the increase in unavailable energy when 1000 Btu are added to a system at constant temperatures of 500°R, 1000°R, and 2000°R, assuming a reference temperature of 400°R.

Solution:

At 500°R

$$\Delta Q_u = 400^{\circ}R \, \frac{1000 \text{ Btu}/^{\circ}R}{500} = 800 \text{ Btu}$$

At 1000°R

$$\Delta Q_u = 400^{\circ}R \, \frac{1000 \text{ Btu}/^{\circ}R}{1000} = 400 \text{ Btu}$$

At 2000°R

$$\Delta Q_u = 400^{\circ}R \, \frac{1000 \text{ Btu}/^{\circ}R}{2000} = 200 \text{ Btu}$$

These processes are represented on the T-S plane in Fig. 4.

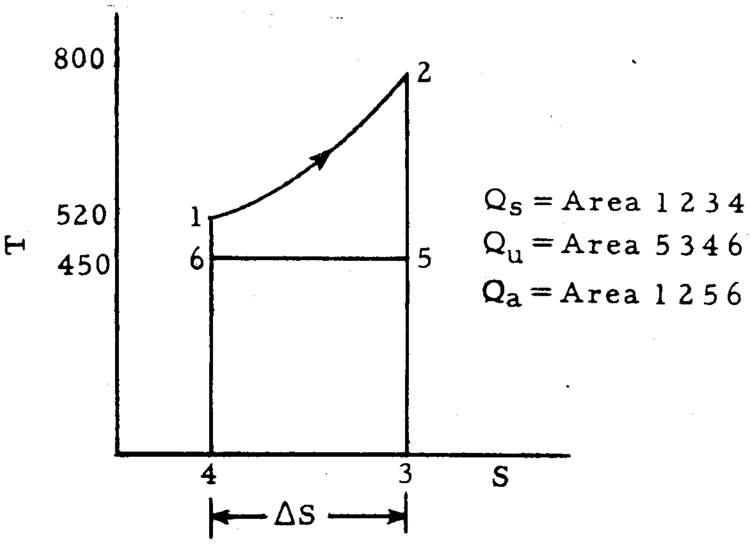
Example 15:

One lb water is heated at constant pressure from 60°F to 340°F. Determine the quantity of heat supplied, the amount of heat supplied that is unavailable for doing useful work based upon a reference temperature of -10°F, and the amount of the heat supplied that is available for doing useful work. Plot the process on a T-s diagram and indicate the various quantities of heat designated.

Solution:

The quantity of heat supplied is:

$$Q_s = c_p(t_2 - t_1) = 1.0 \text{ Btu/lb}^{\circ}F \times (340 - 60)^{\circ}F = 280 \text{ Btu/lb}$$



Increase in entropy as water is heated

Fig. 5

The change in entropy of the water is obtained by using equation (7); thus:

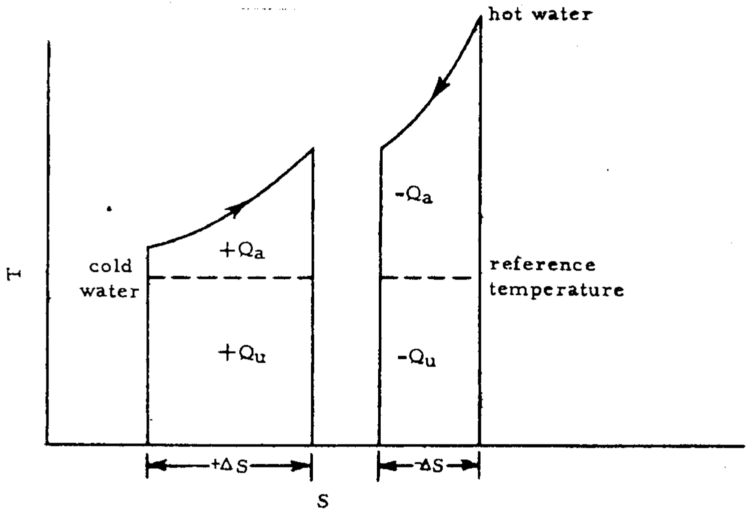
$$\Delta s = \int_{520}^{800} c_p \frac{dT}{T} = 1 \text{ Btu/lb } {}^\circ\text{R} \times \ln \frac{800}{520} = +0.43 \text{ Btu/lb } {}^\circ\text{R}$$

The portion of the energy supplied that is unavailable, or the quantity of unavailable energy, by equation (13) is:

$$Q_u = T_u \Delta s = 450^\circ\text{R} \times 0.43 \text{ Btu/lb}^\circ\text{R} = 193 \text{ Btu/lb}$$

The quantity of available energy is:

$$Q_a = Q_s - Q_u = 280 \text{ Btu/lb} - 193 \text{ Btu/lb} = 87 \text{ Btu/lb}$$



Change in entropy when hot water is mixed with cold

Fig. 6

Example 16:

Ten lb water at $240^{o}F$ are mixed with 10 lb water at $140^{o}F$ under a constant pressure of 30 psia. Determine the change in available and unavailable energies when the temperatures are equalized. Use a reference temperature of $40^{o}F$ and a constant pressure specific heat of 1 Btu/lb^{o}F.

Solution:

The final temperature is determined first.

$$10 \text{ lb} \times 1 \text{ Btu/lb}^{o}\text{F} \times (240 - t_f)^{o}\text{F} = 10 \text{ lb} \times 1 \text{ Btu/lb}^{o}\text{F}$$
$$\times (t_f - 140)^{o}\text{F}; \; t_f = 190^{o}\text{F}$$

The hot water will be considered first.

$$\text{Heat transferred} = wc_y(T_2 - T_1) = 10 \text{ lb} \times 1 \text{ Btu/lb}^{o}\text{R}$$
$$\times (650 - 700)^{o}\text{R} = -500 \text{ Btu}$$

$$\text{Change in entropy} = w \int c_p \frac{dT}{T} = 10 \text{ lb} \times 1 \text{ Btu/lb}^{o}\text{R} \times \int_{700}^{650} \frac{dT}{T}$$
$$= 10 \; (\text{Btu/}^{o}\text{R}) \ln \frac{650}{700} = -0.74 \text{ Btu/}^{o}\text{R}$$

Change in unavailable energy

$$\Delta S \, T_u = -0.74 \text{ Btu/}^{o}\text{R} \times 500 = -370 \text{ Btu (decrease)}$$

Change in available energy

$$= Q_s - Q_u = -500 \text{ Btu} - (-370 \text{ Btu}) = -130 \text{ Btu (decrease)}$$

The cold water is now considered.

$$\text{Heat transferred} = wc_p(T_2 - T_1) = 10 \text{ lb} \times 1 \text{ Btu/lb}^{o}\text{R}$$
$$\times (650 - 600)^{o}\text{R} = +500 \text{ Btu}$$

$$\text{Change in entropy} = w \int c_p \frac{dT}{T} = 10 \text{ lb} \times 1 \text{ Btu/lb}^{o}\text{R} \int_{600}^{650} \frac{dT}{T}$$
$$= 10 \text{ Btu/}^{o}\text{R} \ln \frac{650}{600} = +0.795 \text{ Btu/}^{o}\text{R}$$

Change in unavailable energy

$$= \Delta ST_u = +0.795 \times 500 = +398 \text{ Btu (increase)}$$

Change in available energy

$$= Q_s - Q_u = +500 \text{ Btu} - 398 \text{ Btu} = +102 \text{ Btu (increase)}$$

Net change in available energy = +102 Btu - 130 Btu = -28 Btu (decrease)

Net change in unavailable energy = +398 Btu - 370 Btu = 28 Btu (increase)

Net change in entropy = $+0.795$ Btu/$^\circ$R - 0.74 Btu/$^\circ$R = $+0.055$ Btu (increase)

The change in entropy of the two quantities of water is presented in Fig. 6.

ASSIGNMENT

1. One lb water is heated at constant pressure from 40°F to 140°F. Determine the change in entropy and the increase in unavailable energy when the reference temperature is 0°F.
2. The latent heat of vaporization of water at 30 psia is 945.3 Btu/lb, and the boiling point is 250.33°F. Determine the change in entropy, the portion of the energy supplied that is available energy, and the portion of the energy supplied that is unavailable, referred to a reference temperature of 40°F.
3. Thirty lb water at 240°F are mixed with 30 lb water at 140°F at a constant pressure of 45 psia. Using a reference temperature of 40°F, determine the changes in available energy, unavailable energy, and entropy.
4. Twenty lb steam at 60 psia and 600°F are cooled at constant pressure to 140°F by transferring heat to 244.78 lb water at 40°F. Using the steam tables for all necessary data, determine:

   (a) the quantity of heat transferred,

   (b) the net change in entropy,

4. (c) the change in available energy, and

   (d) the change in unavailable energy based upon a reference temperature of 40°F.

5. Determine the change in entropy when 2 lb moles of oxygen at 1 atm pressure are mixed with 2 moles of nitrogen at 1 atm pressure, both gases having a temperature of 60°F.

6. What is the efficiency of a Carnot engine operating between 60°F and 800°F?

George Howey

Heat and Thermodynamics

MIXTURES OF IDEAL GASES AND VAPOURS

At relatively low pressures the pure gas constituents of a mixture may be considered ideal gases; therefore, the mixture may be treated as a single ideal gas. The following equations apply to such a mixture. Subscripts A, B, C, etc., indicate quantities evaluated for component gases A, B, C, etc.

Dalton's partial pressure law:

$$p_{mix} = p_A + p_B + p_C + \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Partial pressures p_A, p_B, p_C, etc., are evaluated at the temperature and volume of the mixture.

Amagot's partial volume law:

$$V_{mix} = V_A + V_B + V_C + \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Partial volumes V_A, V_B, V_C, etc., are evaluated at the temperature and total pressure of the mixture.

Example 1

Determine the total pressure of a gas mixture composed of 10 lb oxygen, 15 lb nitrogen, and 5 lb hydrogen at 140 F when the total volume is 200 ft3.

Solution

The partial pressures are calculated by using the ideal gas equation.

$$p_{O_2} = \frac{wRT}{V} = 10 \text{ lb} \times 48.25 \text{ ft-lb/lb R} \times \frac{600 \text{ R}}{200 \text{ ft}^3} = 1448 \text{ psf}$$

$$p_{N_2} = 15 \text{ lb} \times 54.99 \text{ ft-lb/lb R} \times \frac{600 \text{ R}}{200 \text{ ft}^3} = 2470 \text{ psf}$$

$$p_{H_2} = 5 \text{ lb} \times 765.86 \text{ ft-lb/lb R} \times \frac{600 \text{ R}}{200 \text{ ft}^3} = 11{,}500 \text{ psf}$$

By equation (1)

$$p_{mix} = (1448 + 2470 + 11{,}500) \text{ psf} = 15{,}418 \text{ psf, or } 106 \text{ psia}$$

Example 2

Determine the total volume of a gaseous mixture at 140 F and 106 psia total pressure, composed of 10 lb oxygen, 15 lb nitrogen, and 5 lb hydrogen.

Solution

The partial volumes will be computed by using the ideal gas equation, then the total volume by using equation (2):

$$V_{O_2} = \frac{wRT}{p} = \frac{10 \text{ lb} \times 48.25 \text{ ft-lb/lb R} \times 600 \text{ R}}{106 \times 144 \text{ lb/ft}^2} = 18.8 \text{ ft}^3$$

$$V_{N_2} = \frac{wRT}{p} = \frac{15 \text{ lb} \times 54.99 \text{ ft-lb/lb R} \times 600 \text{ R}}{106 \times 144 \text{ lb/ft}^2} = 32.2 \text{ ft}^3$$

$$V_{H_2} = \frac{wRT}{p} = \frac{5 \text{ lb} \times 765.86 \text{ ft-lb/lb R} \times 600 \text{ R}}{106 \times 144 \text{ lb/ft}^2} = 149 \text{ ft}^3$$

$$V_{mix} = (18.8 + 32.2 + 149) \text{ ft}^3 = 200 \text{ ft}^3$$

This answer agrees with the original value of example 1.

The mass of the mixture is equal to the sum of the masses of the components; it follows, therefore, that the number of pound moles of the mixture is equal to the sum of the pound moles of components; thus:

$$n_{mix} = n_A + n_B + n_C + \ldots \qquad (3)$$

where n represents the number of pound moles.

When analyzing the effects of a gas mixture process, it is often convenient to use one pound mole of mixture and mole fractions, defined by equation (4), of components, rather than the total moles of gas involved.

$$X_i \text{ (mole fraction of component i)} = \frac{n_i}{n_{mix}} \qquad (4)$$

$$X_A + X_B + X_C + \ldots = 1 \qquad (5)$$

The apparent or equivalent molecular weight of a gas mixture is given by equation (6):

$$M_{mix} = X_A M_A + X_B M_B + X_C M_C + \ldots \qquad (6)$$

An equivalent gas constant R for a mixture may be obtained from the universal gas constant R_o by using the equivalent gas molecular weight.

$$R_{mix} = \frac{R_o}{M_{mix}} \qquad (7)$$

Example 3

Repeat example 1, using (a) the universal gas constant and (b) the mixture gas constant.

Solution (a) The number of moles of each component is determined first.

$$n_{O_2} = \frac{\text{total weight of gas}}{\text{molecular weight}} = \frac{10 \text{ lb}}{32 \text{ lb/lb mole}} = 0.313 \text{ lb mole}$$

$$n_{N_2} = \frac{15 \text{ lb}}{28 \text{ lb/lb mole}} = 0.536 \text{ lb mole}$$

$$n_{H_2} = \frac{5 \text{ lb}}{2 \text{ lb/lb mole}} = 2.5 \text{ lb moles}$$

Using equation (3) gives:

$$n_{mix} = (0.313 + 0.536 + 2.5) \text{ lb moles} = 3.349 \text{ lb moles of mixture}$$

The pressure is computed by using equation (7) lesson 4:

$$p = \frac{nR_oT}{V} = \frac{3.349 \text{ lb mole} \times 1544 \text{ ft-lb/lb mole R} \times 600 \text{ R}}{200 \text{ ft}^3} = 15{,}500 \text{ psf}$$

(b) The component moles fractions are obtained by using equation (4):

$$X_{O_2} = \frac{0.313 \text{ lb mole}}{3.349 \text{ lb mole}} = 0.094; \quad X_{N_2} = \frac{0.536 \text{ lb mole}}{3.349 \text{ lb mole}} = 0.159$$

$$X_{H_2} = \frac{2.50 \text{ lb mole}}{3.349 \text{ lb mole}} = 0.747$$

The equivalent molecular weight is obtained by using equation (6):

$$M_{mix} = (0.094 \times 32 + 0.159 \times 28 + 0.747 \times 2) \text{ lb/lb mole} = 8.95 \text{ lb/lb mole}$$

By equation (7):

$$R_{mix} = \frac{1544 \text{ ft-lb/lb mole R}}{8.95 \text{ lb/lb mole}} = 173 \text{ ft-lb/lb R}$$

Total weight of mixture = (10 + 15 + 5) lb = 30 lb

The ideal gas equation is used to obtain the total pressure:

$$p = \frac{30 \text{ lb} \times 173 \text{ ft-lb/lb R} \times 600 \text{ R}}{200 \text{ ft}^3} = 15{,}500 \text{ psf}$$

Values obtained in parts (a) and (b) agree with the value obtained in example 1.

A combination of equations (1), (2), (3), and the ideal gas equation results in the two important equations given below relating the partial pressure, mole fraction, and total pressure; and the partial volume, mole fraction, and total volume.

$$p_i = X_i \, p_{mix} \quad \text{..............................(8)}$$

$$V_i = X_i \, V_{mix} \quad \text{..............................(9)}$$

These equations are applicable to ideal gas mixtures only. Real gas mixtures are treated in lesson 6.

Example 4

Determine (a) the partial pressures and (b) the partial volume of the components of a gaseous mixture at 200 psia and 240 F containing 20 lb nitrogen and 15 lb oxygen.

Solution (a) The number of moles of each component is determined first.

$$n_{N_2} = \frac{20 \text{ lb}}{28 \text{ lb/lb mole}} = 0.715 \text{ lb mole}$$

$$n_{O_2} = \frac{15 \text{ lb}}{32 \text{ lb/lb mole}} = 0.469 \text{ lb mole}$$

Total number of pound moles n = (0.715 + 0.469) lb mole

= 1.184 lb mole.

The mole fractions are determined by equation (4):

$$X_{N_2} = \frac{0.715 \text{ lb mole}}{1.184 \text{ lb mole}} = 0.604; \quad X_{O_2} = \frac{0.469 \text{ lb mole}}{1.184 \text{ lb mole}} = 0.396$$

The partial pressures are obtained by equation (8):

$$p_{N_2} = 0.604 \text{ x } 200 \text{ psia} = 121 \text{ psia}; \quad p_{O_2} = 0.396 \text{ x } 200 \text{ psia} = 79 \text{ psia}$$

(b) The partial volumes may be computed by using the ideal gas equation:

$$V_{N_2} = \frac{nR_oT}{p} = \frac{0.715 \text{ lb mole x } 1544 \text{ ft-lb/lb mole R x } 700 \text{ R}}{200 \text{ x } 144 \text{ psf}} = 26.9 \text{ ft}^3$$

$$V_{O_2} = \frac{nR_oT}{p} = \frac{0.469 \text{ lb mole x } 1544 \text{ ft-lb/lb mole R x } 700 \text{ R}}{200 \text{ x } 144 \text{ psf}} = 17.6 \text{ ft}^3$$

The total volume, by equation (2) is:

$$V_{mix} = (26.9 + 17.6) \text{ ft}^3 = 44.5 \text{ ft}^3$$

The partial volumes may be checked by using equation (9):

$$V_{N_2} = (0.604 \text{ x } 44.5) \text{ ft}^3 = 26.9 \text{ ft}^3;$$

$$V_{O_2} = (0.396 \text{ x } 44.5) \text{ ft}^3 = 17.6 \text{ ft}^3$$

The other physical properties of a gaseous mixture such as heat capacity, enthalpy, etc. may be computed from the summation of similar component properties by the following equations. Mixture properties are based upon one pound of mixture.

$$c_{p_{mix}} = c_{pA}\frac{w_A}{w_{mix}} + c_{pB}\frac{w_B}{w_{mix}} + c_{pC}\frac{w_C}{w_{mix}} + \ldots\ldots\ldots\ldots (10)$$

$$h_{mix} = h_A \frac{w_A}{w_{mix}} + h_B \frac{w_B}{w_{mix}} + h_C \frac{w_C}{w_{mix}} + \ldots\ldots\ldots\ldots\ldots (11)$$

$$u_{mix} = u_A \frac{w_A}{w_{mix}} + u_B \frac{w_B}{w_{mix}} + u_C \frac{w_C}{w_{mix}} + \ldots\ldots\ldots\ldots\ldots (12)$$

where w represents the weight of a component or the total weight of the mixture according to the subscript.

$$C_{p_{mix}} = C_{pA}X_A + C_{pB}X_B + C_{pC}X_C + \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

$$C_{v_{mix}} = C_v d_A{}^A\ X_A + C_v d_B\ X_B + C_v C\ X_C + \ldots\ldots\ldots\ldots\ldots (13b)$$

$$U_{mix} = U_A\ X_A + U_B\ X_B + U_C\ X_C + \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$H_{mix} = H_A\ X_A + H_B\ X_B + H_C\ X_C + \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Ideal gas mixtures undergoing processes may be treated as single-component gases by employing the mixture properties. The following example will illustrate this method of treatment.

Example 5

A gas mixture composed of 5 lb nitrogen and 4 lb oxygen at 15 psia and 140 F is compressed isentropically to a pressure of 50 psia. Determine, (a) the final temperature, (b) the final volume, (c) the change in internal energy, (d) the change in enthalpy, and (e) the work of compression.

Solution

The equivalent mixture properties are determined first.

The number of moles of each component is:

$$n_{N_2} = \frac{5\ \text{lb}}{28\ \text{lb/lb mole}} = 0.178\ \text{lb mole}$$

$$n_{O_2} = \frac{4\ \text{lb}}{32\ \text{lb/lb mole}} = 0.125\ \text{lb mole}$$

$$n = (0.178 + 0.125)\ \text{lb mole} = 0.303\ \text{lb mole}$$

The mole fractions are obtained by using equation (4):

$$X_{N2} = \frac{0.178\ \text{lb mole}}{0.303\ \text{lb mole}} = 0.59; \qquad X_{02} = \frac{0.125\ \text{lb mole}}{0.303\ \text{lb mole}} = 0.41$$

Equation (6) is used to compute the equivalent molecular weight:

$$M_{mix} = (0.59 \times 28 + 0.41 \times 32)\ \text{lb/lb mole} = 29.6\ \text{lb/lb mole}$$

By equation (7):

$$R_{mix} = \frac{1544\ \text{ft-lb/lb mole R}}{29.6\ \text{lb/lb mole}} = 52.2\ \text{ft-lb/lb R}$$

The specific heats are computed by using equation (10) and a similar equation for the constant volume specific heat.

$$c_{p_{mix}} = \left[0.247\left(\frac{5}{9}\right) + 0.217\left(\frac{4}{9}\right)\right]\ \text{Btu/lb F} = 0.234\ \text{Btu/lb F}$$

$$c_{v_{mix}} = \left[0.176\left(\frac{5}{9}\right) + 0.155\left(\frac{4}{9}\right)\right]\ \text{Btu/lb F} = 0.167\ \text{Btu/lb F}$$

The mixture specific heat ratio is $k_{mix} = \dfrac{c_{p_{mix}}}{c_{v_{mix}}} = \dfrac{0.234\ \text{Btu/lb F}}{0.167\ \text{Btu/lb F}}$

$= 1.40$

The initial total volume is obtained by using the ideal gas equation.

$$V_1 = \frac{wRT}{p} = \frac{9 \text{ lb} \times 52.2 \text{ ft-lb/lb R} \times 600 \text{ R}}{15 \times 144 \text{ lb/ft}^2} = 130 \text{ ft}^3$$

(a) The final temperature is obtained by using the equation given in Table 4 lesson 3 for the isentropic process.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{50}{15}\right)^{\frac{1.40-1}{1.40}} = (3.33)^{0.285} = 1.41$$

$$T_2 = 1.41\ T_1 = 1.41 \times 600 \text{ R} = 845 \text{ R}$$

(b) The final volume is obtained by using the ideal gas equation

$$V_2 = \frac{9 \text{ lb} \times 52.2 \text{ ft/lb-lb R} \times 845 \text{ R}}{50 \times 144 \text{ lb/ft}^2} = 55 \text{ ft}^3$$

(c) The change in internal energy, by equation (19) lesson 4 is

$$\Delta U = wc_v(T_2 - T_1) = 9 \text{ lb} \times 0.167 \text{ Btu/lb R} \times (845 - 600)\text{R}$$

$$= 367 \text{ Btu}$$

(d) The change in enthalpy, by equation (20) lesson 4 is:

$$\Delta H = wc_p(T_2 = T_1) - 9 \text{ lb} \times 0.234 \text{ Btu/lb R } (845 - 600) \text{ R}$$

$$= 516 \text{ Btu}$$

(e) The work of compression may be obtained by application of the first law given in equation (2) lesson 2

$$Q = W + \Delta U; \quad Q = 0 \text{ for isentropic processes}$$

Therefore

$$W = - \Delta U$$

$$W = - 367 \text{ Btu}, \quad \text{or} \quad - 367 \text{ Btu} \times 778 \text{ ft-lb/Btu}$$

$$= -285{,}000 \text{ ft/lb}$$

The minus sign indicates work input to the system. This value may be checked by the following equation obtained from Table 3 lesson 4 for an isentropic process.

$$\text{Work} = \frac{p_2V_2 - p_1V_1}{1 - k}$$

$$= \frac{(50 \times 144) \text{ lb/ft}^2 \times 55 \text{ ft}^3 - (15 \times 144) \text{ lb/ft}^2 130 \text{ ft}^3}{1 - 1.40}$$

$$= - 285{,}000 \text{ ft-lb}$$

Gravimetric and Volumetric Analysis

The analysis of a gaseous mixture may be on a volume or weight basis. When the total pressure is relatively low, permitting the mixture to be treated as an ideal gas, the per cent by volume of a constituent is equivalent to the mole fraction multiplied by 100. The method of changing from one basis of analysis to another is illustrated by examples 6 and 7.

Example 6

Determine the volumetric analysis of a gaseous mixture having the gravimetric analysis: 25% oxygen, 5% carbon dioxide, and 70% nitrogen.

Solution

One hundred lb of mixture will be used; thus there will be 25 lb O_2, 5 lb CO_2, and 70 lb N_2 in 100 lb of mixture.

$$\text{Number of moles of } N_2 = n_{N2} = \frac{70 \text{ lb}}{28 \text{ lb/lb mole}} = 2.5 \text{ lb mole}$$

$$\text{Number of moles of } O_2 = n_{O_2} = \frac{25 \text{ lb}}{32 \text{ lb/lb mole}} = 0.781 \text{ lb mole}$$

$$\text{Number of moles of } CO_2 = n_{CO_2} = \frac{5 \text{ lb}}{44 \text{ lb/lb mole}} = 0.113 \text{ lb mole}$$

Total number of moles, by equation (3) is:

$$n = (0.781 + 0.113 + 2.5) \text{ lb mole} = 3.394 \text{ lb moles}$$

The mole fractions of the components are computed by equation (4)

$$X_{O_2} = \frac{n_{O2}}{n_{mix}} = \frac{0.781 \text{ lb mole}}{3.394 \text{ lb mole}} = 0.231$$

$$X_{N_2} = \frac{2.5 \text{ lb mole}}{3.394 \text{ lb mole}} = 0.736$$

$$X_{CO_2} = \frac{0.113 \text{ lb mole}}{3.394 \text{ lb mole}} = 0.033$$

Per cent by volume = mole fraction x 100; thus:

Per cent by volume of O_2 = 0.231 x 100 = 23.1%
Per cent by volume of CO_2 = 0.333 x 100 = 3.3%
Per cent by volume of N_2 = 0.736 x 100 = 73.6%

Example 7

Determine the gravimetric analysis of a gaseous mixture having the volumetric analysis: 23.1% oxygen, 3.3% carbon dioxide, and 73.6% nitrogen.

Solution

The per cent by volume may be taken as the number of pound moles of components per 100 lb moles of mixture; therefore in 100 moles of mixture there are 23.1 moles of O_2, 3.3 moles of CO_2, and 73.6 moles of N_2.

The number of pounds of component = lb mole of component x molecular weight of component.

$$\begin{aligned}
\text{Pounds of } O_2 &= 23.1 \text{ lb mole} \times 32 \text{ lb/lb mole} = 740 \text{ lb} \\
\text{Pounds of } CO_2 &= 3.3 \text{ lb mole} \times 44 \text{ lb/lb mole} = 145 \text{ lb} \\
\text{Pounds of } N_2 &= 73.6 \text{ lb mole} \times 28 \text{ lb/lb mole} = 2060 \text{ lb}
\end{aligned}$$

Total weight per 100 moles of mixture is 2945 lb

$$\text{Per cent of } O_2 = \frac{\text{lb } O_2}{\text{total weight}} \times 100 = \frac{740 \text{ lb}}{2945 \text{ lb}} \times 100 = 25\%$$

$$\text{Per cent of } CO_2 = \frac{\text{lb } CO_2}{\text{total weight}} \times 100 = \frac{145 \text{ lb}}{2945 \text{ lb}} \times 100 = 4.95\%$$

$$\text{Per cent of } N_2 = \frac{\text{lb } N_2}{\text{total weight}} \times 100 = \frac{2060 \text{ lb}}{2945 \text{ lb}} \times 100 = \underline{70.0\%}$$

$$99.95\%$$

These values check the original gravimetric analysis of example 6.

Mixtures of Air and Water Vapour

At low pressures vapours exhibit the characteristics of ideal gases and may be so treated. Generally the vapour in the mixture is superheated, and in the case of water vapour is assumed to have an enthalpy equal to that of saturated water vapour at the same temperature. When the vapour is in the saturated state the mixture is termed a saturated mixture. Several common terms and equations for air-water vapour mixtures are given in the following:

Relative humidity,

$$\varphi = \frac{\text{partial pressure of water vapour}}{\text{saturation pressure of water vapour at the same temperature}} = \left(\frac{p_w}{p_{sat}}\right)_T \quad \ldots.(16)$$

OR

$$p_w = \varphi \, p_{sat}$$

Applying the ideal gas equation to equation (16), it follows that

$$v_w = \frac{v_{sat}}{\varphi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(17)$$

Specific humidity ω or humidity ratio

$$HR = \frac{\text{mass of water vapour}}{\text{mass of dry air in same volume}} \quad \ldots\ldots\ldots\ldots\ldots\ldots.(18)$$

Usually this is expressed as

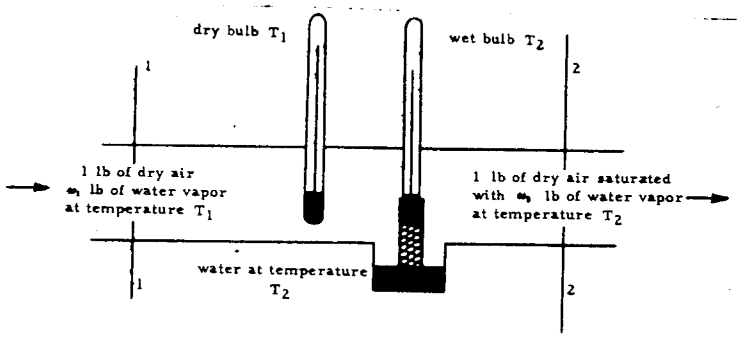
$$\omega = \frac{\text{lb or grains of water vapour}}{\text{1 lb dry air}}$$

Dew Point

A mixture of superheated water vapour and air cooled at constant total pressure, and therefore constant partial water vapour pressure, until it becomes saturated is said to have reached its dew point. The relative humidity at the dew point is 100 per cent since the partial pressure of water vapour p_w is equal to the saturation pressure p_{sat}.

Wet Bulb Temperature

A convenient device used in the determination of the moisture content of an air-water vapour mixture is termed the wet-bulb psychrometer. It consists of two thermometers, one termed the dry-bulb, and the other the wet-bulb since its bulb is covered with a cloth wick saturated with water. An air stream moving past the wet bulb becomes saturated with the moisture evaporated from the wet-bulb wick, thus reducing the temperature of the wet-bulb thermometer. This difference in temperature between the wet- and dry-bulb thermometers is termed the wet-bulb depression. The process described above is termed a process of adiabatic saturation, and for water vapour the wet-bulb temperature is approximately equal to the temperature of adiabatic saturation. This is not true of all vapours. Fig. 1 illustrates the process.



Wet bulb psychrometer

Fig. 1

The following equation may be developed by applying the steady flow general energy equation between sections 1 and 2 of Fig. 1.

$$\omega_1 = \frac{0.24\ (t_2 - t_1) + \omega_2 h_{fg2}}{h_{g1} - h_{f2}} \qquad \text{.....................(19)}$$

where subscript 1 indicates quantities evaluated at dry-bulb temperature, subscript 2 indicates quantities evaluated at wet-bulb temperature, and other subscripts correspond to those used in the steam tables.

An equation was developed relating the partial pressure of the water vapour and the dry- and wet-bulb temperatures. Subscripts 1 and 2 refer to dry- and wet-bulb temperatures, respectively.

$$p_{w_1} = p_{sat_2} - \frac{(p_{mix} - p_{sat_2})(t_1 - t_2)}{2800 - 1.3\,t_2} \quad\ldots\ldots\ldots\ldots(20)$$

The enthalpy of an air-water vapour mixture is usually based upon 1 pound of dry air plus the associated water vapour, but for convenience the enthalpy of the air is based upon 0 F, while that of the water vapour, following the steam table practice, is based upon 32 F. The enthalpy of the mixture is sometimes referred to as the total heat content. The following equation may be used to calculate the enthalpy of the air-water vapour mixture.

$$h = 0.24\ (t - 0) + \omega h_{gt} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

where t is the dry-bulb temperature in degrees F, h_{gt} is the enthalpy of saturated water vapour at the dry-bulb temperature, and ω is the specific humidity of the mixture.

The following examples illustrate the use of the preceding equations.

Example 8

Determine (a) the dew point and (b) the humidity ratio of air at 14.696 psia and 85 F, having a relative humidity of 70%.

Solution

(a) The saturation pressure and specific volume of water vapour at 85 F are obtained from the steam tables.

$$p_{sat} = 0.5959 \text{ psia;} \qquad v_{sat} = 543.5 \text{ ft}^3/\text{lb}$$

The partial pressure, by equation (16) is:

$$p_w = 0.70 \times 0.5959 = 0.417 \text{ psia}$$

The saturation temperature corresponding to this pressure is the dew point, which is obtained from the steam tables.

$$t_{\text{dew point}} = 74 \text{ F}$$

$$p_a = p_{mix} - p_w = 14.696 - 0.417 = 14.279 \text{ psia}$$

The volume of 1 lb dry air is obtained by using the ideal gas equation.

$$v_a = \frac{wRT}{p} = \frac{1 \text{ lb} \times 53.35 \text{ ft-lb/lb R} \times (85 + 460) \text{ R}}{14.279 \times 144 \text{ lb/ft}^2}$$

$$= 14.13 \text{ ft}^3\text{/lb}$$

The specific volume of the water vapour in the mixture is obtained by using equation (17).

$$v_w = \frac{v_{sat}}{\varphi} = \frac{543.5 \text{ ft}^3\text{/lb}}{0.70} = 776 \text{ ft}^3\text{/lb}$$

$$\text{Water vapour in } 14.13 \text{ ft}^3 = \frac{14.13 \text{ ft}^3}{776 \text{ ft}^3\text{/lb}} = 0.0182 \text{ lb}$$

Equation (18) is used to calculate the humidity ratio.

$$HR = 0.0182 \frac{\text{lb water vapour}/14.13 \text{ ft}^3}{1 \text{ lb dry air}/14.13 \text{ ft}^3} = 0.0182 \frac{\text{lb water vapour}}{\text{lb dry air}}$$

The humidity ratio expressed in grains may be obtained by using the conversion factor 7000 grains/lb; thus:

HR = 0.0182 lb water vapour/lb dry air x 7000 grains/lb = 127 grain
water vapour/lb dry air

Example 9

Air at a pressure of 14.969 psia has a dry-bulb temperature of 80 F and a wet-bulb temperature of 65 F. Determine

(a) the specific humidity, (b) the partial pressure of the water vapour, (c) the dew point, (d) the relative humidity, and (e) the enthalpy.

Solution

(a) The following items necessary for the calculation of the specific humidity using equation (19) are obtained from the steam tables.

At 80 F: $h_{sat} = 1096.6$ Btu/lb

At 65 F: $h_{fg} = 1057.1$ Btu/lb; $h_f = 33.05$ Btu/lb

$v_g = 1021$ ft³/lb; $p_{sat} = 0.3056$ psia.

At wet-bulb temperature:

$$p_a = 14.696 - 0.306 = 14.390 \text{ psia}$$

$$v_a = \frac{1 \text{ lb} \times 53.3 \text{ ft-lb/lb R} \times (65 + 460) \text{ R}}{14.39 \times 144 \text{ lb/ft}^2} = 13.5 \text{ ft}^3\text{/lb dry air}$$

$$\omega_2 = \frac{13.5 \text{ ft}^3\text{/lb dry air}}{1021 \text{ ft}^3\text{/lb water vapour}} = 0.0132 \frac{\text{lb water vapour}}{\text{lb dry air}}$$

By equation (19):

$$\omega_1 = \frac{0.24\,(65 - 80) + 0.0132 \times 1057.1}{1096.6 - 33.05} = \frac{0.0097 \text{ lb water vapour}}{\text{lb dry air}}$$

or

$$\frac{0.0097 \text{ lb water vapour}}{\text{lb dry air}} \times \frac{7000 \text{ grains}}{\text{lb}} = \frac{68 \text{ grains water vapour}}{\text{lb dry air}}$$

(b) The partial pressure of the water vapour is determined by using equation (20):

$$p_{w1} = 0.3056 - \frac{(14.696 - 0.3056)(80 - 65)}{2800 - 1.3 \times 65} = 0.226 \text{ psia}$$

(c) The dew-point temperature is obtained from the steam tables corresponding to a pressure of 0.226 psia.

$$t_{\text{dew point}} = 57 \text{ F}$$

(d) Equation (16) is used to calculate the relative humidity. From the steam tables at 80 F, $p_{sat} = 0.5069$ psia. From part (b), $p_W = 0.226$ psia.

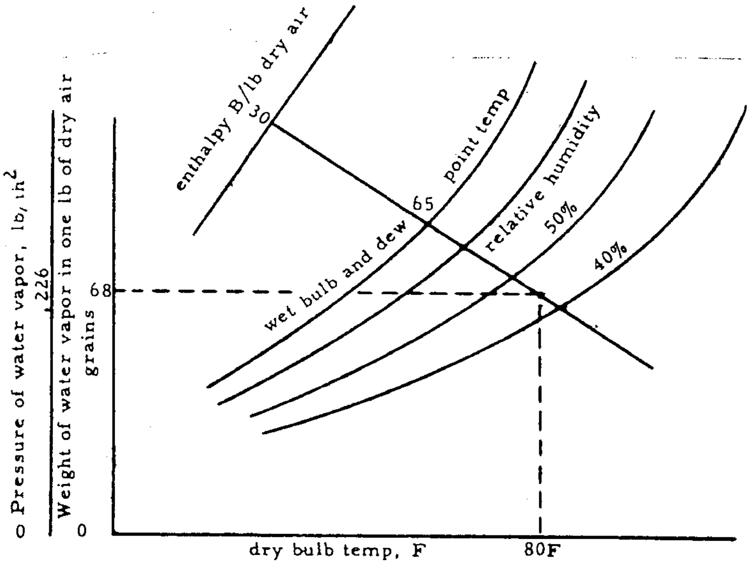
$$\varphi = \frac{0.226}{0.5069} \times 100 = 44.5\%$$

(e) By equation (21):

$$h = 0.24 \times 80 + 0.0097 \times 1096.6 = 29.9 \text{ Btu/lb dry air}$$

Psychrometric Charts

Psychrometric charts have been devised to facilitate the determination of the various air-water vapour properties. Fig. 2 is a skeleton psychrometric chart. The subject of psychrometry will be treated more fully in a following lesson. The following example illustrates the use of the chart in Fig. 2.



Skeleton psychrometric chart

Fig. 2

Example 10

Repeat example 9, using the psychrometric chart.

Solution

(a) Moving horizontally across the chart from the intersection of the 80 F dry-bulb and 65 F wet-bulb temperature lines to the weight of water vapour scale, read the specific humidity or humidity ratio of 68 grains of water vapour/lb dry air.

(b) Continue moving to the left from the weight of water vapour scale to the vapour pressure scale and read the partial pressure of water vapour of 0.226 psia.

(c) Locate the intersection of the 80 F dry-bulb temperature and 65 F wet-bulb temperature lines. Move horizontally across the chart, following the line of constant partial pressure to the saturation curve, and read the dew point temperature of 57 F.

(d) At the intersection of the 80 F dry-bulb and 65 F wet-bulb temperature lines read the relative humidity of 44.5%.

(e) From the intersection of the 80 F dry-bulb and 65 F wet-bulb temperature lines, follow the wet-bulb temperature line to the total heat content scale and read the enthalpy of 30 Btu/lb dry air.

Note that these answers are in agreement with those obtained in example (9).

Air-Water Vapour Processes

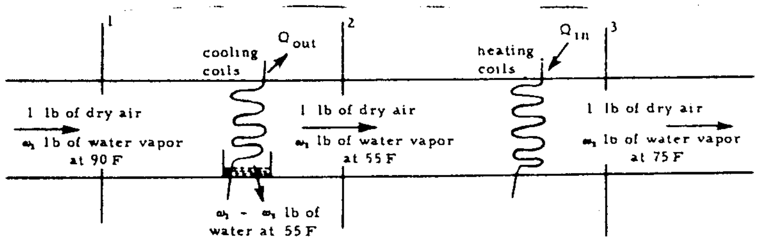
The following examples illustrate typical problems involving moist air.

Example 11

Cooling, dehumidifying, and heating. Determine (a) the amount of moisture removed, (b) the heat removed, and (c) the amount of heat added when 50,000 ft^3 of atmospheric air per hour at 14.696 psia having a dry-bulb temperature of 90 F and a wet-bulb temperature of 80 F is conditioned to air at 75 F having a relative humidity of 50%.

Solution

The problem will be solved by using the steam tables, although the psychrometric chart could be used. For simplification it will be assumed that the mixture is cooled to the dew point of the final mixture, then heated to the final state. Fig. 3 illustrates the process.



Airconditioning process

Fig. 3

(a) The initial and final specific humidities are calculated first. From the steam tables, at 80 F

$$v_{sat} = 633.1 \text{ ft}^3/\text{lb}; \qquad h_{fg} = 1048.6 \text{ Btu/lb}$$

$$h_f = 48.02 \text{ Btu/lb}; \qquad p_{sat} = 0.5069 \text{ psia}$$

at 90 F

$$h_g = 1100.9 \text{ Btu/lb}$$

At the initial wet-bulb temperature

$$p_a = 14.696 - 0.5069 = 14.19 \text{ psia}$$

$$v_a = \frac{1 \text{ lb x } 53.35 \text{ ft-lb/lb R x } (80 + 460) \text{ R}}{14.19 \text{ x } 144 \text{ lb/ft}^2} = 14.15 \text{ ft}^3/\text{lb}$$

$$\omega = \frac{14.15 \text{ ft}^3/\text{lb dry air}}{633.1 \text{ ft}^3/\text{lb water vapour}} = 0.0223 \ \frac{\text{lb water vapour}}{\text{lb dry air}}$$

By equation (19):

$$\omega_1 = \frac{0.24\ (80 - 90) + 0.0223 \text{ x } 1048.6}{1100.9 - 48.02} = 0.020 \ \frac{\text{lb water vapour}}{\text{lb dry air}}$$

At the final state $t = 75$ F; $\varphi = 50\%$. From the steam tables at 75 F:

$$p_{sat} = 0.4298 \text{ psia}; \qquad v_g = 740 \text{ ft3/lb}; \qquad h_g = 1094.5 \text{ Btu/lb}$$

The final partial pressure is obtained by using equation (16).

$$p_w = 0.50 \times .4298 = 0.215 \text{ psia}; \quad p_a = 14.696 - 0.215$$
$$= 14.481 \text{ psia}$$

$$v_a = \frac{1 \text{ lb} \times 53.35 \text{ ft-lb/lb R} \times (460 + 75) \text{ R}}{14.48 \times 144 \text{ lb/ft}^2} = 13.7 \text{ ft}^3\text{/lb}$$

Using equation (17):

$$v_w = \frac{740}{0.50} = 1480 \text{ ft}^3\text{/lb}$$

$$\omega_2 = \frac{13.7 \text{ ft}3\text{/lb dry air}}{1480 \text{ ft}^3\text{/lb water vapour}} = 0.0092 \frac{\text{lb water vapour}}{\text{lb dry air}}$$

Loss of water vapour per pound of dry air

$$\omega_1 - \omega_2 = 0.020 - 0.0092 = 0.0108 \frac{\text{lb water vapour}}{\text{lb dry air}}$$

The initial partial water vapour pressure is obtained by using equation (20).

$$p_{w_1} = 0.5069 - \frac{(14.696 - 0.5069)(90 - 80)}{2800 - 1.3 \times 80} = 0.454 \text{ psia}$$

$$p_{a1} = 14.696 - 0.454 = 14.242 \text{ psia}$$

$$v_{a1} = \frac{1 \text{ lb} \times 53.35 \text{ ft-lb/lb R} \times (460 + 90) \text{ R}}{14.24 \times 144 \text{ lb/ft}^2} = 14.27 \text{ ft}^3\text{/lb}$$

$$\text{Dry air/50,000 ft}^3 \text{ of original mixture} = \frac{50{,}000 \text{ ft}3}{14.27 \text{ ft}^3\text{/lb}} = 3500 \text{ lb}$$

Total water vapour removed

$$= 3500 \text{ lb dry air} \times 0.0108 \frac{\text{lb water vapour}}{\text{lb dry air}} = 37.8 \text{ lb water vapour}$$

(b) The dew point of the final mixture, the temperature to which the initial mixture theoretically must be cooled to remove the required quantity of moisture, is obtained from the steam tables. At 75 F:

$$p_{sat} = 0.4298; \quad p_w = p_{sat}\,\varphi = 0.4298 \times 0.50 = 0.215 \text{ psia}; \quad t = 55 \text{ F}$$

Equation (21) is used to calculate the enthalpy of the mixture at points 1 and 2 of Fig. 3.

$$h_1 = 0.24 \times 90 + 0.020 \times 1100.9 = 43.6 \text{ Btu/lb dry air}$$

At 55 F

$$h_g = 1085.8 \text{ Btu/lb}; \quad h_f = 23.07$$

$$h_2 = 0.24 \times 55 + 0.0093 \times 1085.8 = 23.4 \text{ Btu/lb dry air}$$

Applying the steady flow general energy equation between sections 1 and 2 of Fig. 3 to obtain the quantity of heat removed gives:

$$h_{1\,in} = Q_{out} + h_{2\,out} - (\omega_1 - \omega_2)h_{f\,out}$$

$$Q_{out} = h_{1\,in} - h_{2\,out} - (\omega_1 - \omega_2)h_{f\,out}$$

$$Q_{out} = 43.6 \text{ Btu/lb dry air} - 23.4 \text{ Btu/lb dry air}$$

$$- 0.0108 \text{ lb water/lb dry air} \times 23.07 \text{ Btu/lb water}$$

$$Q = 20.0 \text{ Btu/lb dry air}$$

Total quantity of heat removed

$$= 20.0 \text{ Btu/lb dry air} \times 3500 \text{ lb dry air/hr} = 70{,}000 \text{ Btu/hr}$$

(c) At 75 F

$$h_g = 1094.5 \text{ Btu/lb}$$

$$h_3 = 0.24 \times 75 + 0.0093 \times 1094.5 = 28.2 \text{ Btu/lb dry air}$$

Applying the steady flow general energy equation between sections 2 and 3 of Fig. 3 to obtain the quantity of heat added gives:

$$h_2 + Q_{in} = h_3$$

$$Q_{in} = h_3 - h_2 = 28.2 \text{ Btu/lb dry air} - 23.4 \text{ Btu/lb dry air}$$

$$= 4.8 \text{ Btu/lb dry air}$$

Total amount of heat added

$$= 4.8 \text{ Btu/lb dry air} \times 3500 \text{ lb dry air/hr} = 16{,}800 \text{ Btu/hr}$$

These answers should be checked by using the psychrometric chart.

Example 12

Cooling tower. A cooling tower is used to cool 200,000 lb/hr of water from 105 F to 80 F. Atmospheric air at 90 F having a relative humidity of 60% enters the tower, and leaves at 100 F with a relative humidity of 97%. Determine the volume of moist air required and the quantity of water lost be evaporation per hour. Fig. 4 illustrates the process.

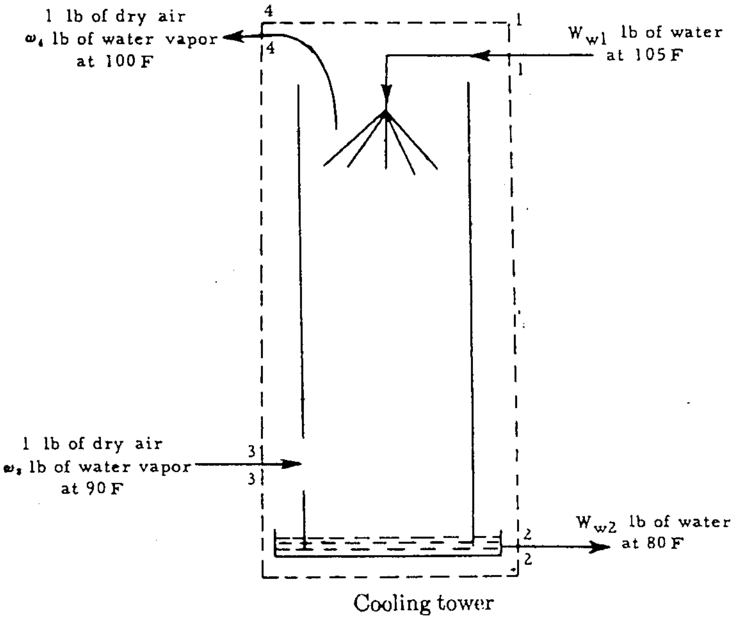


Fig. 4

Solution

One lb dry air will be taken as the basis of analysis. From the psychrometric chart at 90 F, $\varphi = 60\%$.

ω_3 = 0.0183 lb = 128 grains; p_{w3} = 0.42 psia; h_3 = 41.5 Btu/lb dry air

The corresponding values for the air leaving the tower must be calculated, as the point lies outside the range of the psychrometric chart.

From steam tables at 100 F

$$p_{sat} = 0.9492 \text{ psia}; \quad v_g = 350.4 \text{ ft3/lb}; \quad h_g = 1105.2 \text{ Btu/lb}$$

By equation (16):

$$p_w = 0.9492 \text{ x } 0.97 = 0.92 \text{ psia}$$

$$p_{a4} = 14.696 - 0.92 = 13.77 \text{ psia}$$

$$v_{a4} = \frac{1 \text{ lb x } 53.35 \text{ ft-lb/lb R x } (100 + 460) \text{ R}}{13.77 \text{ x } 144 \text{ lb/ft}^2} = 15.1 \text{ ft}^3\text{/lb}$$

By equation (17):

$$v_{w4} = \frac{350.4}{0.97} = 362 \text{ ft3/lb}$$

$$\omega_4 = \frac{15.1 \text{ ft}^3\text{/lb dry air}}{362 \text{ ft3/lb water vapour}} = 0.0417 \text{ lb water vapour/lb dry air}$$

By equation (21):

$$h_4 = 0.24 \text{ x } 100 + 0.0417 \text{ x } 1105.2 = 70.1 \text{ Btu/lb dry air}$$

The amount of water leaving the power per pound of dry air is the amount entering per pound of dry air minus that lost by evaporation; thus:

$$W_{w2} = W_{w1} - (\omega_4 - \omega_3) = W_{w1} - (0.0417 - 0.0183) = W_{w1} - 0.0234$$

The enthalpies of the water are obtained from the steam tables. At 105 F,

$$h_f = 72.95 \text{ Btu/lb; at 80 F, } h_f = 48.02 \text{ Btu/lb}$$

The amount of water entering the tower per pound of dry air is obtained by applying the steady-flow general energy equation to the tower.

$$h_3 + W_{w1}h_f = h_4 + W_{w2}h_{f2}$$

$$41.5 + W_{w1}72.95 = 70.1 + (W_{w1} - 0.0234)\ 48.02$$

$$W_{w1} = \frac{70.1 - 41.5 - 1.13}{72.95 - 48.02} = 1.105 \text{ lb water/lb dry air}$$

Quantity of water lost by evaporation:

$$\omega_4 - \omega_3 = 0.0417 - 0.0183 = 0.0234 \text{ lb water/lb dry air}$$

Partial pressure of the atmospheric air

$$p_a = 14.696 - 0.42 = 14.27 \text{ psia}$$

$$v_{a3} = \frac{1 \text{ lb} \times 53.35 \text{ ft-lb/lb R} \times (460 + 90) \text{ R}}{14.27 \times 144 \text{ lb/ft}^2} = 14.3 \text{ ft}^3\text{/lb dry air}$$

Quantity of dry air required to cool the water:

$$W_{air} = \frac{200{,}000 \text{ pound of water/hr}}{1.105 \text{ lb water/lb dry air}} = 181{,}000 \text{ lb dry air/hr}$$

Atmospheric air required

$$= 14.3 \text{ ft}^3\text{/lb dry air} \times 181{,}000 \text{ lb dry air/hr} = 2{,}590{,}000 \text{ ft}^3\text{/hr}$$

Water lost by evaporation

$$= 0.0234 \text{ lb/lb dry air} \times 181{,}000 \text{ lb dry air/hr} = 4250 \text{ lb/hr}$$

$$\omega_2 = \frac{2.00}{633.1} = 0.00315 \text{ lb water vapour/lb dry air}$$

$$\text{Moisture condensed} = \omega_1 - \omega_2 = 0.0132 - 0.0032$$

$$= 0.010 \text{ lb/lb dry air}$$

ASSIGNMENT

1. A mixture composed of 15 lb nitrogen and 10 lb oxygen at a pressure of 50 psia has a temperature of 60 F. For the mixture determine (a) mole fraction of each component, (b) the gas constant, (c) the equivalent molecular weight, (d) the mixture specific volume, (e) the partial pressures, and (f) the partial volumes.

2. Determine the specific heats of the mixture of problem 2.

3. Determine the volume of a gaseous mixture composed of 14 lb nitrogen, 32 lb oxygen, 56 lb carbon monoxide, and 8 lb hydrogen at 130 F under a pressure of 215 psia.

4. Determine the partial pressure and total pressures of a gaseous mixture resulting from the mixing of 20 ft^3 oxygen at 215 psia and 500 F, and 30 ft^3 nitrogen at 135 psia and 300 F. The final volume of the mixture is 12 ft^3, and the mixture is permitted to cool to 150 F.

5. Determine the gravimetric analysis of a gaseous mixture having the volumetric analysis: 51% N_2, 3% CH_4, 9% CO_2, 22% CO, and 15% H_2.

6. Determine the relative humidity, humidity ratio, and enthalpy for air having a wet-bulb temperature of 60 F and a dry-bulb temperature of 80 F, at a pressure of 14.7 psia. Check your answers by using the psychrometric chart.

7. What are the relative humidity and partial pressure of the water vapour for air having a dry-bulb temperature of 85 F and a wet-bulb temperature of 65 F at atmospheric pressure?

8. Determine the wet-bulb temperature, partial water vapour pressure, relative humidity, humidity ratio, and enthalpy of moist air having a dry-bulb temperature of 90 F, a dew point of 65 F at 1 atm pressure.

9. A 10 ft^3 tank contains moist air at a total pressure of 100 psia at 300 F with a relative humidity of 5%. The mixture is cooled to 80 F. Determine (a) the partial pressures of the original mixture (b) the partial pressures of the final mixture, (c) and the weight of moisture condensed.

10. A building requires 45,000 ft3 air per minute at 75 F and a relative humidity of 40%. Atmospheric air at 92 F with a relative humidity of 60% is used. Calculate the quantity of heat removed and the heat supplied per minute.

Course 1A25
Lesson 1A25.00-9

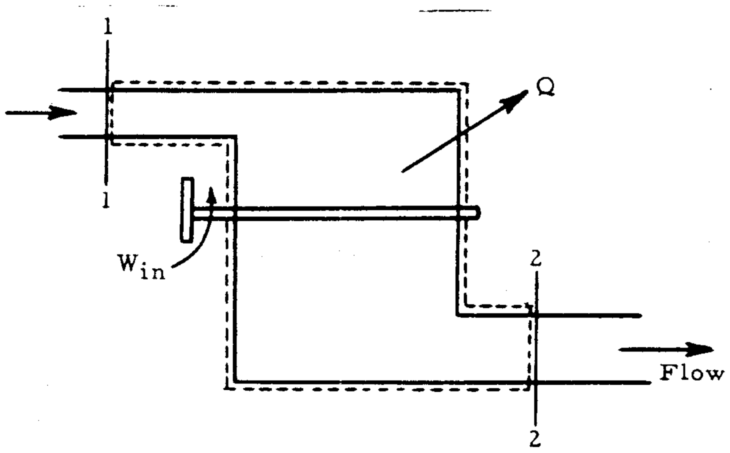
Heat & Thermodynamics

GAS COMPRESSORS AND ENGINES

Gas compressors are used to supply high-pressure gases for use in gas liquefaction processes, for gas turbines, to supply air for operating pneumatic tools, etc. Each of the two general types, rotary and reciprocating, is treated in this lesson. A brief section is devoted to the air engine.

Rotary Compressors

The rotary compressor, centrifugal, axial flow, positive displacement, etc., is in general a high-speed device used to supply large quantities of relative low-pressure gas. Applying the steady-flow energy equation to the compressor as pictured in Fig. 1 gives



Generalized Compressor

Fig. 1

$$Q - \frac{W}{J} = w\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2gJ}\right]$$

in which w is the weight of gas compressed per unit time. There are no elevation terms because of the low specific weight of the gas. This equation when solved for the work becomes:

$$W = -w\left[J\,(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2g}\right] + QJ \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In this equation the sign notation of equation (2) lesson 2 is used; therefore, a negative W indicates work input to the gas, and a negative Q indicates heat transfer from the gas.

When the heat transfer and kinetic energy terms are neglected, a measure of the performance of the compressor is given by the adiabatic efficiency defined as:

$$\eta_{ad} = \frac{\text{isentropic work}}{\text{actual work}} = \frac{\text{isentropic } \Delta h'}{\text{actual } \Delta h} = \frac{h'_2 - h_1}{h_2 - h_1} \quad \ldots.(2)$$

The following examples illustrate the use of the preceding equations.

Example 1

A centrifugal compressor having an adiabatic efficiency of 85% is used to compress 150 lb nitrogen per min from 15 psia and 60 F to 45 psia. Determine the horsepower required to drive the compressor, neglecting changes in kinetic energy and heat transfer.

Solution

Assuming nitrogen to be a perfect gas:

$$\Delta h' = c_p \Delta t' = c_p (T_2' - T_1)$$

From Table 3 lesson 4 for isentropic compression

$$\frac{T'_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

The value of k for nitrogen, from Table 1 lesson 4, is 1.4.

$$T_2' = (60 + 460) \text{ R} \left(\frac{45}{15}\right)^{\frac{1.4-1}{1.4}} = 710 \text{ R}$$

The value of c_p for nitrogen, from Table 1 lesson 4, is 0.247 Btu/lb R. The isentropic work of compression, by equation 1, is:

$$\frac{W'}{J} = -150 \text{ lb/min} \times 0.247 \text{ Btu/lb R} \times (710 - 520) \text{ R}$$

$$= 7050 \text{ Btu/min}$$

The actual work of compression by equation (2) is:

$$\frac{W}{J} = -\frac{7050 \text{ Btu/min}}{0.85} = -\ 8300 \text{ Btu/min}$$

$$\text{Work} = -\ 8300 \text{ Btu/min} \times \frac{1 \text{ hp}}{42.5 \text{ Btu/min}} = -\ 195.5 \text{ hp}$$

Example 2

An axial flow compressor having an adiabatic efficiency of 87% is used to compress 200 lb air per min from 14.696 psia and 60 F to 55 psia. Determine the horsepower required to drive the compressor, neglecting changes in kinetic energy and heat transfer.

Solution

From the air tables of the Keenan and Kaye Gas Tables the enthalpies $h_1 = 124.27$ and $h_2' = 181.5$ are obtained as explained in example 12 lesson 4. The isentropic work of compression, by equation (1), is:

$$\frac{W'}{J} = -\ 200 \text{ lb/min} \left[(181.5 - 124.27) \text{ Btu/lb}\right] = -\ 11{,}446 \text{ Btu/min}$$

The actual work of compression, by equation (2) is:

$$W = \frac{-11{,}440 \text{ Btu/min}}{0.87} \times \frac{1}{42.4 \text{ Btu min/hp}} = -\ 310 \text{ hp}$$

It is customary to neglect the negative sign.

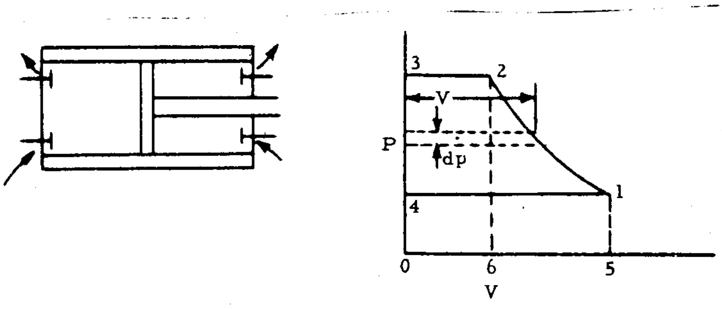
A different form of equation (1) for an ideal gas with constant specific heat, assuming isentropic compression and neglecting the change in kinetic energy is:

$$W = -\frac{k}{k-1} \times wRT_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] = \frac{k}{1-k} p_1 v_1$$

$$\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] \quad \ldots\ldots\ldots\ldots\ldots(3)$$

Reciprocating Compressors Without Clearance

The reciprocating compressor may be treated as a steady-flow device, and the steady-flow energy equation may be applied, as was done in the section on Rotary Compressors; however, the influence of several important factors upon its operation is more evident from a stepwise analysis. For simplicity the compressor is assumed to be single-acting, and the change in kinetic energy of the entering and leaving gas is neglected.



Reciprocating compressor and p-V diagram

Fig. 2

From Fig. 2 it can be seen that, starting point 4, the work exchanged between the piston and gas during one complete reversible cycle is equal to suction work W_1, represented by area 4150, plus polytropic compression work W_2, represented by area 1265, plus discharge work W_3 represented by area 2306. These work terms can be evaluated by using the equations for work for the polytropic and constant-pressure processes from Table 3 lesson 4.

$$W_1 = p_1(V_1 - V_4); \qquad W_2 = \frac{p_2V_2 - p_1V_1}{1 - n}; \qquad W_3 = p_2(V_3 - V_2)$$

Adding these work terms to obtain the net cycle work, noting that $V_4 = V_3 = 0$, gives:

$$W = W_1 + W_2 + W_3 = p_1V_1 + \frac{p_2V_2 - p_1V_1}{1 - n} - p_2V_2 = \frac{n}{1 - n}(p_2V_2 - p_1V_1) \ldots\ldots (4)$$

It should be noted that this net work represented by areas 1234 of Fig. 2 may also be evaluated by $\int Vdp$.

Since the cyclic work is negative (work performed upon the system consisting of the gas), the expression becomes:

$$W = -\int_{p_1}^{p_2} Vdp \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

A more convenient expression for the work of compression in terms of the pressure ratio p_2/p_1 may be obtained by multiplying and dividing equation (4) by p_1V_1 and noting that for the polytropic process

$$V_2/V_1 = (p_2/p_1)^{-1/n}$$

The resulting expression is:

$$W = \frac{n}{1 - n} p_1V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad \text{.........(6)}$$

Equation (6), when applied to an ideal gas, may be written in terms of the initial temperature by substituting wRT_1 for p_1V_1.

An expression for the cyclic work of compression for an isentropic compression may be obtained by substituting k for n in equation (6).

When the compression is assumed to be isothermal, $p_1V_1 = p_2V_2$, and the work during compression W_2 is $p_1V_1 \ln V_2/V_1$. The cycle work using isothermal compression is:

$$W = p_1V_1 \ln \frac{V_2}{V_1} \quad \text{..........................(7)}$$

It should be noted that all equations in this section neglect kinetic energy effects.

Example 3

Ten lb air per min are compressed from 14.696 psia and 60 F to 60 psig when the barometer is 14.696 psia. Determine the horsepower required to drive the compressor, assuming (a) isothermal compression, (b) isentropic compression, and (c) polytropic compression with n = 1.30.

Solution

(a) The initial volume of 10 lb air is obtained by using the ideal gas equation.

$$V_1 = 10\text{ lb} \times 53.35\text{ ft-lb/lb R} \times \frac{(60 + 460)\text{ R}}{144 \times 14.696\text{ psf}} = 131.0\text{ ft}^3$$

The final pressure was given as gauge pressure; therefore the final absolute pressure is:

$$p_2 = (14.696 + 60) = 74.696 \text{ psia}$$

The final volume, obtained by the ideal gas equation, is:

$$V_2 = 10 \text{ lb} \times 53.35 \text{ ft-lb/lb R} \times \frac{(60 + 460)\text{R}}{144 \times 74.696 \text{ psf}} = 25.8 \text{ ft}^3$$

Substituting the above values into equation (7) gives:

$$W = (14.696 \times 144)\frac{\text{lb}}{\text{ft}^2} \times 131 \text{ ft}^3 \times \ln \frac{25.8 \text{ ft}^3}{131.0 \text{ ft}^3} = -450{,}000 \text{ ft-lb}$$

This represents the work per minute; therefore the horsepower is:

$$W = -450{,}000 \text{ ft-lb/min} \times \frac{1 \text{ hp}}{33{,}000 \text{ ft-lb/min}} = -13.6 \text{ hp}$$

(b) Substituting the appropriate values from part (a) into equation (6) with n = k gives:

$$W = \frac{1.4}{1 - 1.4}(144 \times 14.696)\frac{\text{lb}}{\text{ft}^2} \times 131 \text{ ft}^3 \left[\left(\frac{74.696}{14.696}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$= -572{,}000 \text{ ft-lb/min, or } -17.3 \text{ hp}$$

(c) Substituting the appropriate values into equation (6) with n = 1.3 gives:

$$W = \frac{1.30}{1 - 1.30}(144 \times 14.696)\frac{\text{lb}}{\text{ft}^2} \times 131 \text{ ft}^3 \left[\left(\frac{74.696}{14.696}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= -546{,}000 \text{ ft-lb/min, or } -16.6 \text{ hp}$$

Comparison of these results indicates a saving of 3.7 hp by complete cooling, and 0.70 hp by partial cooling.

Example 4

Work part (c) of example (3), using equation (1).

Solution

The final temperature is obtained by using the polytropic relation

$$T_2/T_1 = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

from Table 3, lesson 4

$$T_2 = 520\ \text{R} \left(\frac{74.696}{14.696}\right)^{\frac{1.3-1}{1.3}} = 757\ \text{R}$$

The polytropic specific heat is:

$$c_n = c_v \left(\frac{n-k}{n-1}\right) = 0.173\ \text{Btu/lb R}\ \frac{1.3 - 1.40}{1.3 - 1} = -\ 0.0578\ \text{Btu/lb R}$$

The negative sign indicates heat removal.

Equation (1) written for 10 lb air, assumed to be an ideal gas, and neglecting kinetic energies, is:

$$W = -10\ \frac{\text{lb}}{\text{min}}\ \left[0.24\ \text{Btu/lb R}\ (757 - 520)\ \text{R}\right] + 10\ \text{lb/min} \times (-0.0578)(757 - 520)$$

$$= -\ 568\ \text{Btu/min} - 137\ \text{Btu/min} = -\ 705\ \text{Btu/min} = -\ 16.6\ \text{hp}$$

This value agrees with that obtained in part (c) of example 3.

Reciprocating Compressors with Clearance

In the actual compressor, part of the gas remains in the cylinder at the end of the discharge stroke, and expands as the piston starts upon its intake stroke. The volume occupied by this gas is termed clearance volume, V_c of Fig. 3, and is specified as a percentage of the piston displacement V_D; thus:

$$\text{Clearance} = C = \frac{V_c}{V_D} \times 100 \qquad \text{.................(8)}$$

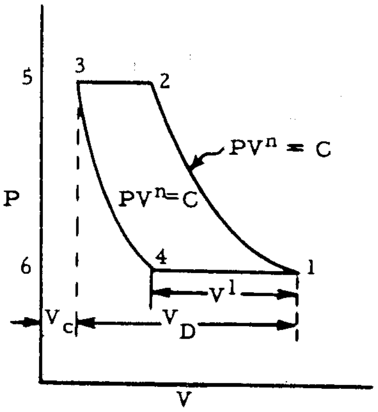
Clearance volume theoretically does not affect the cyclic work of compression. Actually the effect of friction and the larger required displacement volume with clearance increases the net work of the cycle.

Another term used to indicate the effect of clearance is called clearance factor, defined in terms of Fig. 3 as:

$$\text{Clearance factor} = CF = \frac{V'}{V_D} = \frac{V_1 - V_4}{V_D} \qquad \text{(C)}$$

$$V_4 = V_3 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = CV_D \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$V_1 = V_D + CV_D$$



Theoretical P-V diagram with clearance

Fig. 3

Substituting the above values into equation (C) gives:

$$CF = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \qquad \text{.........................(9)}$$

Example 5

A compressor having an 8% clearance is used to compress nitrogen from 20 to 75 psia. What is the clearance factor when n = 1.19?

Solution

Substituting the clearance value into equation (9) gives:

$$CF = 1 + 0.08 - 0.08\left(\frac{75}{20}\right)^{\frac{1}{1.19}} = 0.84 \text{ or } 84\%$$

Example 7

A compressor having a clearance of 7% is used to compress 15 lb carbon dioxide from 20 psia and 75 F to 70 psia. Assuming a value of n equal to 1.15, determine (a) the work required, (b) the piston displacement required with no clearance, and (c) the piston displacement required with clearance.

Solution

(a) The value of R for CO_2 obtained from Table 1 lesson 4 is 35.09 ft-lb/lb R. Substituting the appropriate values into equation (6) gives:

$$W = \frac{1.15}{1-1.15} \times 15 \text{ lb} \times 35.09 \text{ ft-lb/lb R } (75+460)\text{R} \left[\left(\frac{70}{20}\right)^{\frac{1.15-1}{1.15}} - 1\right] = -383{,}000 \text{ ft-lb}$$

(b) The total volume of gas which is equal to the piston displacement without clearance, as obtained by using the ideal gas equation, is:

$$V_D = V' = 15 \text{ lb} \times 35.09 \text{ ft-lb/lb R} \left(\frac{535 \text{ R}}{(144 \times 20)\frac{\text{lb}}{\text{ft}^2}}\right) = 98 \text{ ft}^3$$

(c) The clearance factor, by equation (9) is:

$$CF = 1 + 0.07 - 0.07\left(\frac{70}{20}\right)^{\frac{1}{1.15}} = 1.07 - 0.208 = 0.86$$

Equation (C) is used to obtain the piston displacement with clearance. V', the volume of gas compressed, is equal to 98 ft3, from part (b).

$$CF = \frac{V'}{V_D} \quad \text{or} \quad V_D = \frac{V'}{CF} = \frac{98 \text{ ft3}}{0.86} = 114 \text{ ft}^3$$

In the actual compressor the pressure drop through the intake passage and valves causes the pressure in the cylinder to be less than the pressure in the intake system. In the case of air compressors using atmospheric air the intake process is assumed to be isothermal, so that $p_aV_a = p_1V_1$, in which the subscript a indicates atmospheric conditions, and subscript 1 indicates conditions inside the compressor cylinder. The air at prevailing atmospheric conditions is referred to as free air, and a compressor may be rated upon the volume of free air that it can compress per unit of time.

Certain losses such as leakage loss, pressure loss, etc., reduce the weight of gas delivered by a compressor below the theoretical value. The ratio of the actual weight of gas delivered to the weight of gas that would occupy the piston displacement volume under intake conditions is defined as the volumetric efficiency of the compressor.

$$\eta_v = \frac{\text{weight of gas discharged per cycle}}{\text{weight of gas that would occupy piston displacement under intake conditions per cycle}} \quad \ldots\ldots(10)$$

$$\eta_v = \frac{\text{Volume of gas measured at intake conditions entering compressor per cycle}}{\text{Piston displacement volume per cycle}} \quad \ldots\ldots\ldots(10a)$$

Under ideal conditions with no losses.

$$\eta_v = \frac{V'}{V_D} = \text{clearance factor}$$

Several other efficiencies used in the compressor field are defined as follows:

Mechanical efficiency =

$$\eta_M = \frac{\text{indicated horsepower of compressor}}{\text{brake horsepower supplied}} \quad \ldots\ldots\ldots\ldots\ldots(11)$$

Adiabatic efficiency =

$$\eta_{ad} = \frac{\text{isentropic work of compression}}{\text{actual adiabatic work of compression}} \quad \ldots\ldots\ldots\ldots\ldots(12)$$

Isothermal efficiency =

$$\eta_{is} = \frac{\text{isothermal reversible work of compression}}{\text{actual isothermal work of compression}} \qquad ......(13)$$

The following example illustrates the application of several of the preceding equations.

Example 7

An air compressor having 5% clearance is used to compress 900 ft3 of free air per min to a pressure of 75 psia. The atmospheric pressure is 14.696, and the pressure within the cylinder at the beginning of the compression stroke is 14.0 psia. Assuming a value of n equal to 1.3, determine (a) the theoretical horsepower required to drive the compressor, (b) the cylinder volume when the compressor is double-acting, running at 200 rpm and (c) the actual horsepower required to drive the compressor if its mechanical efficiency is 92%.

Solution

(a) Since clearance does not affect the theoretical work of compression, equation (6) may be used. In the absence of V_1, p_aV_a will be substituted for p_1V_1; thus:

$$W = \frac{1.30}{1 - 1.30} 14.696 \times 144 \text{ lb/ft}^2 \times 900 \text{ ft}^3\text{/min} \left[\left(\frac{75}{14.696} \right)^{\frac{1.30 - 1}{1.30}} - 1 \right]$$

$$= - 3{,}770{,}000 \text{ ft-lb/min, or } - 114 \text{ hp}$$

(b) For a compressor with clearance, V' is equal to the volume of air in the cylinder at pressure p_1. Assuming isothermal intake:

$$p_aV_a = V'p_1, \quad \text{or} \quad V_a = \frac{V'p_1}{p_a}$$

The volumetric efficiency, be equation (10a), is $n_v = V_a/V_D$. Substituting the previous value of V_a gives $n_v = p_1V'/p_aV_D$; but $V'/V_D = CF$; therefore $n_v = (p_1/p_a)CF$. Substituting the appropriate values into this expression gives:

$$\eta_v = \frac{14.0}{14.696}\left[1 + 0.05 - 0.05\left(\frac{75}{14}\right)^{\frac{1}{1.3}}\right] = 0.83$$

The compressor is double-acting and makes 200 rpm; therefore the number of cycles per minute is 2 x 200, or 400. The volume of air at inlet conditions per cycle is equal to the total volume of free air per minute divided by the total cycles per minute, or

$$V' = \frac{900\ \text{ft}^3/\text{min}}{400\ \text{cycles/min}} = 2.25\ \text{ft}^3/\text{cycle}$$

The piston displacement is obtained by using equation (10a).

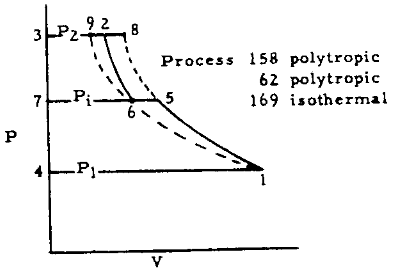
$$\eta_v = 0.83 = \frac{V'}{V_D} = \frac{2.25}{V_D}, \quad \text{or} \quad V_D = \frac{2.25}{0.83} = 2.71\ \text{ft}^3$$

(c) The actual power required to drive the compressor as determined by equation (11) is:

$$\eta_M = 0.92 = \frac{114\ \text{hp}}{\text{bhp}}, \quad \text{or} \quad \text{bhp} = \frac{114}{0.92} = 124\ \text{hp}$$

Multistage Compression

As the discharge pressure is increased, the discharge temperature increases and may reach prohibitive values. It should also be noted that as p_3 of Fig. 3 is increased, other values remaining constant, the point 4 will move further to the right, thus materially reducing the volume of new gas received per stroke. These and other practical considerations have given rise to the practice of multistaging with intercooling between stages. In the 2-stage compressor the gas is compressed to some intermediate pressure in the low-pressure cylinder, discharged and cooled at constant pressure, then compressed to the discharge pressure in the high-pressure cylinder. The theoretical cycle without clearance is illustrated in Fig. 4. The gas is



Theoretical P–V diagram of two stage compressor without clearance and with no pressure drop in intercooler assumed to be cooled to the initial temperature in this analysis, and no pressure loss in the intercooler is considered. The work of the complete cycle, assumed to take place in two separate compressors, by equation (6), is:

$$W = \frac{n}{1 - n} p_1 V_1 \left[\left(\frac{p_i}{p_1} \right)^{\frac{n - 1}{n}} - 1 \right] + \frac{n}{1 - n} p_i V_6 \left[\left(\frac{p_2}{p_i} \right)^{\frac{n - 1}{n}} - 1 \right]$$

But $p_i V_6 = p_1 V_1$ by reason of the complete intercooling; therefore

$$W = \frac{n}{1 - n} p_1 V_1 \left[\left(\frac{p_i}{p_1} \right)^{\frac{n - 1}{n}} + \left(\frac{p_i}{p_1} \right)^{\frac{n - 1}{n}} - 2 \right]$$

Differentiating this expression with respect p_i and equating to zero enables one to determine the value of p_i for minimum work. The resulting value is:

$$p_i = \sqrt{p_1 p_2}$$

which, when substituted into the cyclic work expression, gives:

$$W = \frac{2n}{1 - n} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n - 1}{2n}} - 1 \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

This indicates that the cyclic work becomes a minimum when the work is equally divided between the stages.

The net saving in work by intercooling is indicated in Fig. 4 by the area 5826. The intermediate pressures for minimum work in a 3-stage compressor are:

p_i 1st stage = $\sqrt[3]{p_1^2\ p_2}$; p_i 2nd stage = $\sqrt[3]{p_1 p_2^2}$.

A general expression for the cyclic work of a multistage compressor having X stages is:

$$W = \frac{Xn}{1 - n}\, p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{Xn}} - 1 \right] \ldots\ldots\ldots\ldots\ldots (15)$$

In the actual compressor, as a result of pressure loss in the passages, valves, intercooler, etc., the intake pressure of the second cylinder is less than the discharge pressure of the preceding cylinder; for this reason equation (6) should be used for each cylinder rather than equation (15) for the complete compressor.

Example 8

A 3-stage compressor is to be used to compress air from 15 psia and 70 F to 600 psia, with a value of n = 1.32. Compute for the theoretical cycle (a) the intermediate pressures, (b) the cyclic work of compression per pound of air, (c) the temperatures leaving each cylinder, (d) the heat transfer in each intercooler, and (e) the total work by the steady flow analysis.

Solution

(a) The intermediate pressures are determined as follows:

$$P_i \text{ 1st stage} = \sqrt[3]{(15)^2(600)} = 51.3 \text{ psia}$$

$$P_i \text{ 2nd stage} = \sqrt[3]{(15)(600)^2} = 175.2 \text{ psia}$$

(b) The value of p_1V_1 of equation (15) may be replaced by wRT_1, where w represents the pounds of gas discharged; therefore the cyclic work, by equation (15), is:

$$W = \frac{3 \times 1.32}{1 - 1.32} \times (1 \text{ lb}) \times 53.35 \text{ ft-lb/lb R} \times (70 + 460) \text{ R}$$

$$\left[\left(\frac{600}{15}\right)^{\frac{1.32-1}{3\,(1.32)}} - 1\right] = -\ 121{,}500 \text{ ft/lb}$$

(c) The temperatures leaving the cylinders, by the polytropic expression from Table 3, lesson 4, is:

$$T \text{ leaving first stage} = (70 + 460)\text{R}\left(\frac{51.3}{15}\right)^{\frac{1.32-1}{1.32}} = 715 \text{ R}$$

The temperature leaving the intercooler and entering the second stage is also 530 R; therefore

$$T \text{ leaving the second stage is } 530 \text{ R}\left(\frac{175.2}{51.3}\right)^{\frac{1.32-1}{1.32}} = 715 \text{ R}$$

and likewise for the third stage.

$$T \text{ leaving the third stage is } 530 \text{ R}\left(\frac{600}{175.2}\right)^{\frac{1.32-1}{1.32}} = 715 \text{ R}$$

(d) The heat transfer in the intercooler is:

$$Q = wc_p\,(T_2 - T_1) = 1 \text{ lb} \times 0.24 \text{ Btu/lb R} \times (530 - 715)\text{R} = -\ 44.5 \text{ Btu}$$

Both intercoolers have the same heat transfer since they are operating between the same temperatures.

(e) The polytropic specific heat for the cooling process within the cylinders is:

$$c_n = c_v\left(\frac{n-k}{n-1}\right) = 0.173 \text{ Btu/lb R}\left(\frac{1.32 - 1.4}{1.32 - 1}\right) = -\ 0.0433 \text{ Btu/lb R}$$

The work per cycle per cylinder as obtained, by using equation (1), is:

$$W = 1 \text{ lb} \left[c_p (T_2 - T_1) - c_n (T_2 - T_1)\right] = 1 \text{ lb} \left[0.24 \text{ Btu/lb R} \times (715 - 530) \text{ R} - (-0.0433 \text{ Btu/lb R})(715 - 530)\right] = -52.4 \text{ Btu}$$

The total work for the 3 cylinders is:

$$W = 3 \times (-52.4 \text{ Btu}) = -157 \text{ Btu}, \quad \text{or} \quad 157 \text{ Btu} \times 778 \text{ ft-lb/Btu} = -122{,}000 \text{ ft-lb}$$

which checks closely the value obtained in part (b).

Compressed-Gas Engines

Compressed air and other gases may be expanded in reciprocating engines or in turbines without first being heated. These devices are not classified as prime movers however. Air hammers or drills or examples.

ASSIGNMENT

1. Assuming constant specific heat, determine the horsepower required to compress isentropically 10 lb CO_2 per min from 20 psia and 60 F to 150 psia.

2. Air at standard sea-level condition (60 F, 1 atm) enters an air compressor with the velocity of 400 mph, is isentropically compressed to a pressure of 7 atm, and leaves with a velocity of 300 fps. How much work is required per pound of air, assuming constant specific heat?

3. A single-stage air compressor without clearance is used to compress 3.75 lb air per min from 70 F and 14.7 psia to 150 psia. Compute the horsepower required if n for the polytropic compression has a value of 1.35.

4. Determine the theoretical horsepower required to compress isothermally 6.13 lb air per min from 60 F and 14.7 psia to 120 psia.

5. Determine the horsepower required and the piston displacement of an air compressor having a clearance of 3% when used to compress 1600 ft^3 free air per min from an atmospheric pressure of 14.7 psia to 100 psia. The pressure before compression is 14.0 psia, and n is assumed equal to 1.3. Assume the compressor to be double-acting, operating at 150 rpm.

6. A centrifugal compressor operating at steady flow and having an adiabatic efficiency of 85% compresses air at 70 F and 14.7 psia, to 100 psia. Determine (a) the work required per pound and (b) the temperature of air leaving the compressor.

George Howey

Heat & Thermodynamics

VAPOUR POWER CYCLES, STEAM ENGINES AND STEAM TURBINES

Carnot Cycle

The Carnot cycle, Fig. 1, using steam as a working fluid is impractical; however, it may be used as a criterion of perfection. The cycle efficiency expressed in enthalpies is:

$$\eta = \frac{\text{work}_{\text{out}} - \text{work}_{\text{in}}}{\text{heat}_{\text{in}}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} =$$

$$1 - \frac{h_4 - h_1}{h_3 - h_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

When expressed in terms of temperature the equation becomes:

$$\eta = 1 - \frac{T_1 \Delta S}{T_2 \Delta S} = 1 - \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1a)$$

Rankine Cycle

The Rankine cycle, Fig. 2, a more practical criterion of perfection for steam cycles, is made up of the following reversible steady-flow processes:

(a) isentropic compression in the pump of the saturated water leaving the condenser

(b) constant-pressure heating and evaporation of the water in the boiler

(c) isentropic expansion of the steam in the prime mover

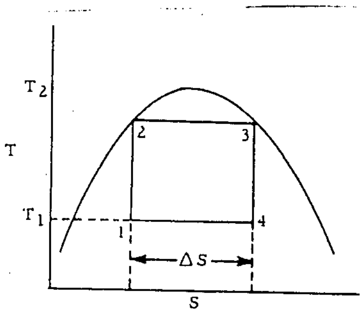
(d) constant-pressure condensation of the steam in the condenser.

Neglecting kinetic energy terms, the thermal efficiency of the Rankine cycle using saturated steam, expressed in enthalpies is:

$$\eta = \frac{\text{work}_{out} - \text{work}_{in}}{\text{heat}_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} \quad \ldots\ldots\ldots(2)$$

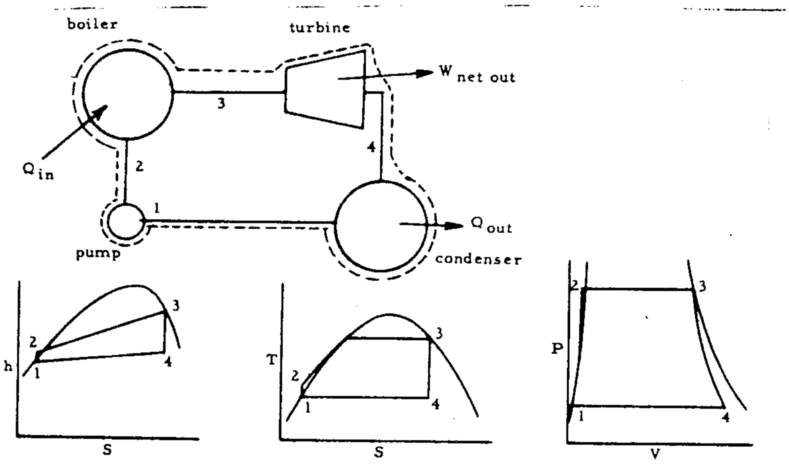
At low pressures, the pump work, being small in comparison to the other quantities, is usually neglected, resulting in equation (2) being written:

$$\eta = \frac{h_3 - h_4}{h_3 - h_2} = \frac{h_3 - h_4}{h_3 - h_{f2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$



Carnot cycle using saturated steam

Fig. 1



Rankine steam cycle using saturated steam

Fig. 2

Example 1:

Determine the efficiency of a Rankine cycle operating with saturated steam at 200 psia when the exhaust pressure of the turbine is 5 psia. Compare the results with the efficiency of a Carnot cycle operating between the same temperatures.

Solution:

The enthalpies are obtained from the steam table. At 200 psia $h_3 = h_g = 1198.4$ Btu/lb, $t = 381.8$ F, $s_g = 1.5453$. The enthalpy h_4 is obtained by isentropic expansion from 200 psia to 5 psia. At 5 psia, $h_f = 130.13$ Btu/lb, $h_{fg} = 1001.0$ Btu/lb, $s_f = 0.2347$, $s_{fg} = 1.6094$. The actual enthalpy h_4 is obtained by first using equation (5) lesson 3 to determine tha quality, and then equation (3) to calculate the enthalpy.

$$s_3 = s_4 = s_{f4} + X_4 s_{fg4} \qquad \text{or}$$

$$X_4 = \frac{s_3 - s_{f4}}{s_{fg4}} = \frac{1.5453 - 0.2347}{1.6094} = 0.815$$

$$h_4 = h_{f4} + X_4 h_{fg4} = 130.13 \text{ Btu/lb} + 0.815 \times 1001 \text{ Btu/lb}$$
$$= 946 \text{ Btu/lb}$$

This value may be checked by using the Mollier diagram. The efficiency as obtained by using equation (3) is:

$$\eta = \frac{1198.4 \text{ Btu/lb} - 946 \text{ Btu/lb}}{1198.4 \text{ Btu/lb} - 130.1 \text{ Btu/lb}} \times 100 = 23.6\%$$

The Carnot cycle efficiency, by equation (1a) is:

$$\eta = 1 - \frac{(162.2 + 460)\text{R}}{(381.8 + 460)\text{R}} \times 100 = 26\%$$

The areas on the T - s diagram represent the heat supplied, and the heat rejected, or unavailable energy; therefore

Solution: (cont'd)

changes in cycle operating conditions are reflected by corresponding changes in areas on the T - s diagram. Changes in a basic Rankine cycle using saturated steam caused by (1) lowing the exhaust pressure, (2) increasing the operating pressure, and (3) using superheated steam are illustrated in example (2).

Example 2:

A basic Rankine cycle uses saturated steam at 300 psia and exhausts at 5 psia. Determine (a) the efficiency of the cycle, (b) the effect upon efficiency of lowering the back pressure to 1 psia (c) the effect upon efficiency of increasing the operating pressure to 400 psia, (d) the effect upon efficiency of using superheated steam at 520 F and 300 psia, and (e) the efficiency of a Rankine cycle operating with superheated steam at 400 psia and 520 F exhausting at 1 psia.

Solution:

The enthalpies are obtained from the steam tables and Mollier diagram as illustrated in example (1).

(a) at 300 psia, h_g = 1202.8 Btu/lb = h_3. Isentropic expansion of saturated steam at 300 psia to wet steam at 5 psia gives h_4 a value of 924 Btu/lb. At 5 psia, h_f = 130.13 Btu/lb. Using equation (3) gives a cycle efficiency of:

$$\eta = \frac{1202.8 \text{ Btu/lb} - 924 \text{ Btu/lb}}{1202.8 \text{ Btu/lb} - 130.13 \text{ Btu/lb}} \times 100 = 25.9\%$$

(b) The isentropic expansion of saturated steam at 300 psia to wet steam at 1 psia gives h_4 a value of 844 Btu/lb. At 1 psia, h_f = 69.7 Btu/lb.

The cycle efficiency, by equation (3) is:

$$\eta = \frac{(1202.8 - 844) \text{ Btu/lb}}{(1202.8 - 69.7) \text{ Btu/lb}} \times 100 = 31.7\%$$

(c) At 400 psia, h_g = 1204.5 Btu/lb. Isentropic expansion of saturated steam at 400 psia to 5 psia gives h_4 a

(c) value of 908 Btu/lb. At 5 psia, h_f = 130.13 Btu/lb. The cycle efficiency is:

$$\eta = \frac{(1204.5 - 908)\text{ Btu/lb}}{(1204.5 - 130.13)\text{ Btu/lb}} \times 100 = 27.6\%$$

(d) At 300 psia and 520 F, h_g = 1269.5 Btu/lb. Isentropic expansion of superheated steam at 520 F and 300 psia to 5 psia gives h_4 a value of 968. h_f at 5 psia is 130.13 Btu/lb. The cycle efficiency is:

$$\eta = \frac{1269.5\text{ Btu/lb} - 968\text{ Btu/lb}}{1269.5\text{ Btu/lb} - 130.1\text{ Btu/lb}} \times 100 = 26.5\%$$

(e) Isentropic expansion of superheated steam from 300 psia and 520 F to 1 psia gives h_4 a value of 882 Btu/lb. h_f at 1 psia = 69.7 Btu/lb. The cycle efficiency is:

$$\eta = \frac{1269.5\text{ Btu/lb} - 882\text{ Btu/lb}}{1269.5\text{ Btu/lb} - 69.7\text{ Btu/lb}} \times 100 = 32.2\%$$

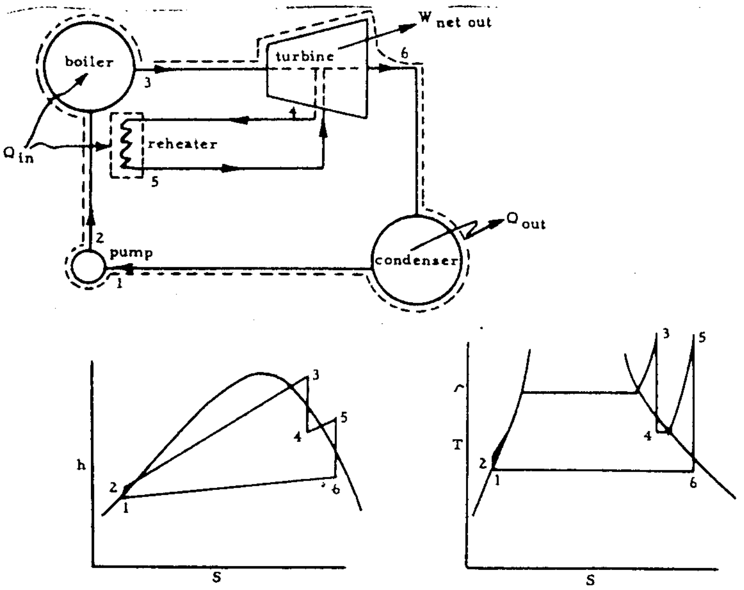
Reheat Cycle

In the reheat cycle, the steam, after partial expansion in the turbine, is withdrawn and resuperheated at constant pressure in the steam generator and then returned to the turbine and completely expanded, as illustrated in Fig. 3. The work output per pound of steam is $(h_3 - h_4) + (h_5 - h_6)$ the pump work is $(h_2 - h_1)$, and the heat supplied is $Q_{in} = (h_3 - h_2) + (h_5 - h_4)$. The cycle efficiency is:

$$\eta = \frac{W_{out} - W_{in}}{Q_{in}} = \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)} \quad \ldots.(4)$$

Example 3:

Determine the efficiency of a reheat cycle operating under the following conditions. Steam initially at 800 psia and 700 F is expanded to 40 psia, resuperheated to 500 F, and exhausted at 3 in. Hg.



Reheat steam cycle

Fig. 3

Solution:

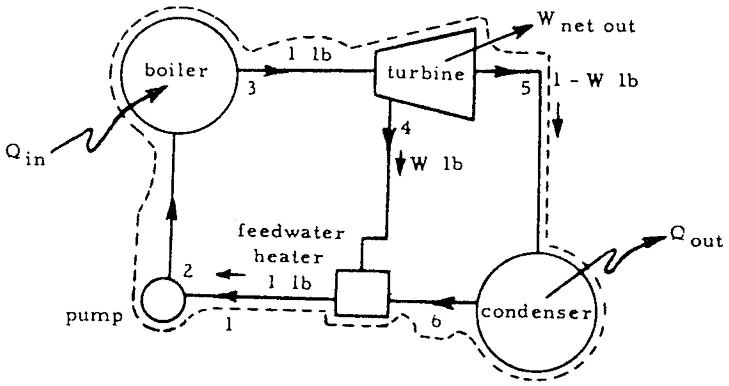
The enthalpies are obtained from the steam tables and Mollier diagram. At 800 psia and 700 F, $h_g = h_3 = 1338.6$ Btu/lb. Isentropic expansion from 800 psia and 700 F to 40 psia gives h_4 a value of 1075 Btu/lb. At 40 psia and 500 F, $h_g = h_5 = 1284.8$ Btu/lb. Isentropic expansion from 40 psia and 500 F to 3 in. Hg gives a value for h_6 of 1036 Btu/lb. At 3 in. Hg, $h_f = h_1 = 82.99$ Btu/lb, and t = 115 F. The value of h_2 is obtained from the compressed liquid tables as follows. At 800 psia and 115 F, $h - h_f = 2.11$ Btu/lb; at 115 F, $h_f = 82.99$. Therefore at 800 psia and 115 F, $h = 82.99 + 2.11 = 85.10$ Btu/lb.

Substituting these values into equation (4) gives:

$$\eta = \frac{(1338.6 - 1075) + (1284.8 - 1036) \quad (85.10 - 82.99)}{(1338.6 - 85.10) + (1284.8 - 1075)} = 34.9\%$$

Regenerative Cycle

The efficiency of a steam cycle may be increased by removing a fraction of the steam, after partial expansion in the turbine, for use in increaseing the temperature of the condensate returning to the steam generator, as illustrated in Fig. 4. The following example illustrates the method of determining the regenerative cycle efficiency.



Regenerative steam cycle

Fig. 4

Example 4:

A regenerative cycle using 1 open feedwater heater expands steam from 400 psia and 780 F to 4 psia. Steam is extracted at 18 psia for feedwater heating. Determine the cycle efficiency.

Solution:

The necessary enthalpies are obtained from the steam tables and Mollier diagram in the following manner. At 400 psia and 780 F, $h_3 = h_g = 1405.8$ Btu/lb. Isentropic expansion from 400 psia and 780 F to 18 psia gives h_4 a value of 1110 Btu/lb, and expanding to 4 psia gives h_5 a value of 1013 Btu/lb. Since the feedwater heater is open, the pressure in the heater is 18 psia, and $h_1 = h_f$ at 18 psia = 190.56 Btu/lb. At 4 psia, $h_6 = h_f = 120.86$ Btu/lb. If the pump work is neglected $h_2 = h_1 = 190.56$ Btu/lb. The weight of steam extracted per pound of steam entering the turbine is obtained by making an energy balance neglecting kinetic energy changes at the feedwater heater.

Energy in Energy out

$$(1 - w)\ \text{lb}\ h_6 + w\ \text{lb}\ h_4 = 1\ \text{lb}\ h_1$$

Solving for w gives:

$$w = \frac{h_1 - h_6}{h_4 - h_6} = \frac{190.56\ \text{Btu/lb} - 120.86\ \text{Btu/lb}}{1110\ \text{Btu/lb} - 120.86\ \text{Btu/lb}} = 0.07\ \text{lb/lb steam}$$

$$\text{Work output} = 1\ \text{lb}\ (h_3 - h_4) + (1 - w)\ \text{lb}\ (h_4 - h_5)$$

Work output/lb steam

$$= 1\ \text{lb}\ (1405.8 - 1110)\ \text{Btu/lb} + (1 - 0.07)\ \text{lb}\ (1110 - 1013)\ \text{Btu/lb}$$

$$= 385.8\ \text{Btu/lb}$$

Heat supplied

$$= Q_{in} = (h_3 - h_2) = (1405.8 - 190.6)\ \text{Btu/lb} = 1215.2\ \text{Btu/lb}$$

The efficiency of the cycle, neglecting pump work, is:

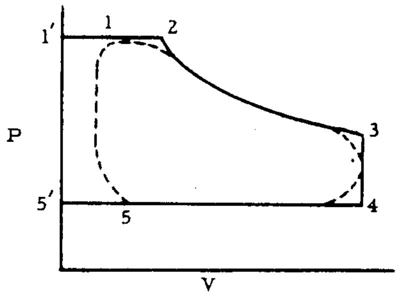
$$\eta = \frac{w_{out}}{Q_{in}} = \frac{385.8\ \text{Btu/lb}}{1215.2\ \text{Btu/lb}} \times 100 = 31.7\%$$

More efficient cycles are obtained by using a greater number of feedwater heaters and by combining the reheat and regenerative cycles. The method of cycle analysis is similar to that illustrated in the previous examples.

Steam Engines

Steam engines find wide application in the small prime mover field especially when the required speed of rotation is low. They have good efficiencies and are extremely reliable.

The steam engine may be analyzed as an intermittent-flow device by using the indicator card, or as a steady-flow device by using enthalpies. An indicator card, a reduced-pressure volume diagram, of the action of the steam in the cylinder, is shown in Fig. 5. The mean effective pressure, mep, or p_m is obtained by dividing the net card area by its length, thus obtaining the average pressure acting upon the piston during 1 cycle. The indicated horsepower, ihp, which represents the energy transferred from the steam to the piston is obtained by the following equation.



Steam engine indicator diagram, ideal and real

Fig. 5

$$\text{ihp} = \frac{p_m LAN}{33{,}000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which p_m = mep in psi

L = length of piston stroke in ft.

A = piston area in in.^{2}

N = power strokes per min

The actual output of the engine may be measured by means of some mechanical, hydraulic, or electric brake, and is expressed as brake horsepower or bhp. The ratio of bhp to ihp is the mechanical efficiency of the engine.

$$\eta \text{ mech} = \frac{\text{bhp}}{\text{ihp}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The steam rate, defined as the pounds of steam per unit of output, is also used to compare the performance of steam engines. If W_B represents the output work of the engine per pound of steam supplied, expressed in Btu the steam rate is:

$$w = \frac{2545 \text{ Btu/hp-hr}}{W_B\text{/lb steam}} = \text{lb steam/hp-hr} \ldots\ldots\ldots\ldots\ldots\ldots (7a)$$

$$w = \frac{3413 \text{ Btu/kw-hr}}{W_B\text{/lb steam}} = \text{lb steam/kw-hr} \ldots\ldots\ldots\ldots\ldots\ldots (7b)$$

Treated as a steady-flow device, the work done per pound of steam by an ideal steam engine, neglecting the kinetic energy, is the difference between the enthalpies of the steam supplied, h_1, and the exhaust steam, h_2' obtained by an isentropic expansion from the initial condition to the exhaust pressure. The quantity of energy supplied per pound of steam is determined by taking the difference between the enthalpy of the steam supplied, h_1, and the enthalpy of saturated water at the exhaust pressure, because the engine is not designed to obtain work from a liquid. The ideal or Rankine engine efficiency is:

$$\eta_R = \frac{h_1 - h_2'}{h_1 - h_{f2}} \quad \dots\dots\dots\dots (8)$$

The work of the actual steam engine per pound of steam is $(h_1 - h_2)$, where h_1 is the enthalpy of the steam supplied, and h_2 is the enthalpy of the exhaust steam. The thermal efficiency is:

$$\eta_{act} = \frac{h_1 - h_2}{h_1 - h_{f2}} \quad \dots\dots\dots\dots (9)$$

or in terms of the steam rate:

$$\eta_{act} = \frac{2545}{w\ (h_1 - h_{f2})} \quad \dots\dots\dots\dots (9a)$$

An efficiency ratio used to compare actual and ideal steam engines, variously called engine efficiency and Rankine cycle ratio, is obtained by dividing the actual thermal efficiency by the Rankine engine efficiency:

$$\eta_e = \frac{\eta_{act}}{\eta_R} \quad \dots\dots\dots\dots (10)$$

The following example illustrates the use of the equation presented in this section:

Example 5:

The following data were obtained during a test of a steam engine.

Cylinder diameter = 12 in.
Stroke = 15 in.
Piston rod diameter = 2.5 in.
Head in card area = 2.7 in.^{2}
Crank in card area = 2.75 in.^{2}
Card length = 3.4 in.
Spring scale = 60 psia/in.

Net brake load = 600 lb
Brake arm radius = 3 ft.
Speed = 250 rpm
Initial steam pressure = 115 psia
Quality of steam = 100%
Exhaust pressure = 15 psia
Steam consumption = 3000 lb/hr

Example 5: (cont'd)

Determine (a) indicated horsepower, (b) brake horsepower, (c) mechanical efficiency, (d) steam rate, (e) actual thermal efficiency, (f) Rankine cycle efficiency, and (g) engine efficiency.

Solution:

(a) The mep's are obtained by the following equation:

$$p_m = \text{mep} = \frac{\text{card area in.}^2}{\text{card length in.}} \text{ x spring scale x lb/in.}$$

$$\text{head end } p_m = \frac{2.7 \text{ in.}^2}{3.4 \text{ in.}} \text{ x } 60 \text{ lb/in.}^3 = 47.7 \text{ psi}$$

$$\text{crank end } p_m = \frac{2.75 \text{ in.}^2}{3.4 \text{ in.}} \text{ x } 60 \text{ lb/in.}^3 = 48.6 \text{ psi}$$

Using equation (5) gives:

$$\text{head end ihp} = \frac{\dfrac{47.7 \text{ lb}}{\text{in.}^2}\left(\dfrac{15}{12}\right)\text{ft x }\left[\dfrac{12^2\,\pi}{4}\right]\text{in.}^2 \text{ x } 250 \text{ rpm}}{33{,}000 \text{ ft-lb/min hp}} = 51.0 \text{ hp}$$

crank end ihp

$$= \frac{\dfrac{48.6 \text{ lb}}{\text{in.}^2} \text{ x } \left(\dfrac{15}{12}\right)\text{ft }\left[\dfrac{12^2\,\pi}{4} - \dfrac{2.5^2\,\pi}{4}\right]\text{in.}^2 \text{ x } 250 \text{ rpm}}{33{,}000 \text{ ft-lb/min hp}} = 50.5 \text{ hp}$$

$$\text{Total ihp} = 50.5 + 51.0 = 101.5 \text{ hp}$$

(b) The brake horsepower is determined by the equation:

(b) (cont'd)

$$\text{bhp} = \frac{2\pi\ \text{RFN}}{33{,}000} = \frac{2 \times 3.14 \times 3 \text{ ft} \times 600 \text{ lb} \times 250 \text{ rpm}}{33{,}000 \text{ ft-lb/min hp}} = 85.6 \text{ hp}$$

(c) The mechanical efficiency is obtained by using equation (6).

$$\eta_{\text{mech}} = \frac{85.6 \text{ hp}}{101.5 \text{ hp}} \times 100 = 84.5\%$$

(d) The steam rate is:

$$w = \frac{\text{lb steam}}{\text{hr (bhp)}} = \frac{3000 \text{ lb}}{85.6 \text{ hp-hr}} = 35 \text{ lb steam/hp-hr}$$

As a check,

$$W_B = \frac{\text{bhp hr} \times 2545 \text{ Btu/hp-hr}}{1 \text{ lb steam/hr}} = \frac{85.6 \text{ bhp} \times 1 \text{ hr} \times 2545 \text{ Btu/hp-hr}}{3000 \text{ lb steam/hr}}$$

$$= 72.8 \text{ Btu/lb steam/hr}$$

Substituting into equation (7a) gives:

$$w = \frac{2545 \text{ Btu/hp-hr}}{72.8 \text{ Btu/lb steam}} = 35 \text{ lb steam/hp-hr}$$

(e) The enthalpy of the steam supplied h_1 from the steam tables at 115 psia is 1189.7 Btu/lb; h_f at 15 psia = 181.1 Btu/lb = h_{f2}. Substituting into equation (9a) gives:

$$\eta_{\text{act}} = \frac{2545 \text{ Btu/hp-hr}}{35 \text{ lb/hp-hr } (1189.7 - 181.1) \text{ Btu/lb}} \times 100 = 7.2\%$$

(f) The enthalpy after isentropic expansion from the entrance condition to the exhaust pressure of 15 psia from the Mollier diagram is 1040 Btu/lb. Substituting the appropriate values into equation (8) gives:

$$\eta_R = \frac{(1189.7 - 1040)\ \text{Btu/lb}}{(1189.7 - 181.1)\ \text{Btu/lb}} \times 100 = 14.9\%$$

(g) The engine efficiency using equation (10) is:

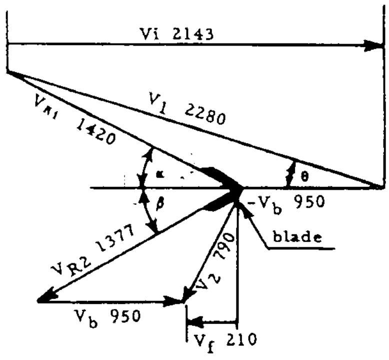
$$\eta_c = \frac{7.2\%}{14.9\%} \times 100 = 48.3\%$$

Steam Turbines

In the steam turbine prime mover, the kinetic energy that is absorbed by moving blades and delivered as shaft work to the device being driven by the prime move originates from the expansion of steam in the nozzles. The turbine is inherently a high-speed device, and has a basic advantage over the steam engine in that its motion is purely rotary. Modern steam turbines vary in size from small units developing a few horsepower to compound units developing over 200,000 horsepower. The two basic types of turbines are impulse and reaction, depending upon their mode of operation. Each of these types will be discussed below.

(a) Impulse Turbine

The simplest impulse turbine has a single stage consisting of a set of stationary nozzles and one row of moving blades. Low-velocity steam entering the stage is expanded in the nozzles to a high velocity, enters the moving blades, is turned through approximately 150 degrees, and then is discharged. The velocity diagram of an actual impulse stage is presented in Fig. 6. The relative velocity of the steam leaving the blade is less than the relative velocity entering, because of fluid friction.



Velocity diagram for impulse stage with blade loss

Fig. 6

The work done by the stage may be obtained by writing the steady-flow energy equation between points a and c for 1 pound of steam.

$$J\ h_a + \frac{V_a{}^2}{2\ g} = J\ h_c + \frac{V_c{}^2}{2\ g} + W, \qquad \text{or}$$

$$W = (h_a - h_c)\ J + \frac{V_a{}^2 - V_c{}^2}{2\ g} \quad \dots\dots\dots\dots\dots\dots(11)$$

A similar equation for the blade is:

$$J\ h_b + \frac{V_b{}^2}{2\ g} = J\ h_c + \frac{V_c{}^2}{2\ g} + W, \qquad \text{or}$$

$$W = (h_b - h_c)\ J + \frac{V_c{}^2 - V_c{}^2}{2\ g} \quad \dots\dots\dots\dots\dots\dots(12)$$

If there are no losses in the blade, $h_c = h_b$.

The work done by the stage, Fig. 6, may be obtained by kinematic analysis as follows. Force applied to the blade per pound of steam per second is:

$$F = ma = (V_{R2} \cos a - V_{R1} \cos\beta)\ \frac{1}{g}$$

The work done per second per unit weight of fluid is:

$$\text{Work} = \text{Force x distance} = (V_{R2} \cos a - V_{R1} \cos\beta)\ \frac{V_b}{g} \quad .(13)$$

or

$$W = (V_f - V_i)\ \frac{V_b}{g} \quad \dots\dots\dots\dots\dots\dots\dots\dots(13a)$$

It should be noted that equation (13) and (13a) are written in terms of vectors; therefore the direction of the velocites must be considered. It can be shown that for maximum work:

$$V_b = 0.5\ V_1 \cos \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The blading efficiency is determined by dividing the work done by the blades by the kinetic energy supplied, thus:

$$\eta = \frac{(V_f - V_i)\ V_b/g}{V_1^2/2g} = \frac{2\ (V_f - V_i)\ V_b}{V_1^2} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

Neglecting the kinetic energies entering and leaving a stage, the stage efficiency is equal to the actual change in enthalpy through the stage divided by the change in enthalpy for an ideal stage having the same pressure drop.

$$\eta = \frac{h_a - h_c}{h_a - h'_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The difference between $(h_a - h'_c)$ and $(h_a - h_c)$ is termed the reheat.

Example 6:

Superheated steam at 62 psia and 311 F enters the nozzles of a symmetrical impulse stage with negligible velocity, and is expanded to 15 psia. The velocity of the blade is 950 fps, the ratio of the relative velocity leaving the blade to the relative velocity entering in 0.97, and the nozzle efficiency is 96%. Determine (a) the work done by the stage and (b) the blade efficiency when the nozzle angle θ is 20°.

Solution:

(a) At 62 psia and 311 F, h_g = 1187.2 Btu. From the Mollier diagram the value of the enthalpy after isentropic expansion to 15 psia is 1081. The velocity V_1 of the steam leaving the nozzle is obtained by using equation (15).

$$V_1 = \sqrt{2 \times 32.2\ \text{ft/sec}^2 \times 778\ \text{ft-lb/Btu} \times (1187.2 - 1081)\ \text{Btu} \times 0.96} = 2260\ \text{fps}$$

(a) The velocity diagram Fig. 6 is drawn to scale, and the other necessary velocities are determined from it. The work is obtained by using equation (13).

$$W = (-1195 - 1160) \text{ ft/sec} \times 950 \text{ ft/sec} \times \frac{1 \text{ lb}}{32.2 \text{ ft/sec}^2}$$

$$= -69{,}500 \text{ ft-lb/lb}$$

The work by equation (13a) is:

$$W = (-210 - 2143) \frac{\text{ft}}{\text{sec}} \times \frac{950 \text{ ft}}{\text{sec}} \times \frac{1 \text{ lb}}{32 \text{ ft/sec}^2}$$

$$= -69{,}500 \text{ ft-lb/lb}$$

The negative sign has no significance here.

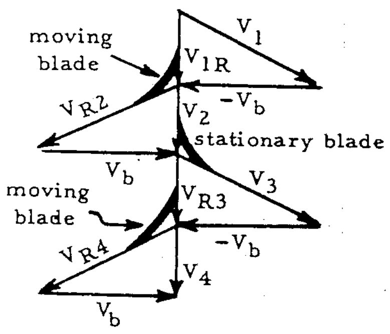
(b) The blading efficiency by equation (15) is:

$$\eta = \frac{(-210 - 2143)}{(2260)^2} \times 2 \times 950 \times 100 = 87\%$$

A combination of one set of nozzles and two moving rows of blades separated by one set of stationary blades is called an impulse velocity compound stage or Curtis stage.

(b) Reaction Stage

In the reaction stage, Fig. 7, the velocity of the steam is increased and the pressure decreased in the moving blades as well as in the nozzles or stationary blades. The increase in the relative velocity at exit over the relative velocity at entrance gives a reaction force to the moving blades.



Reaction Stages

Fig. 7

This principle of operation gave rise to the name reaction turbine. One set of nozzles or stationary blades and one set of moving blades constitute a reaction stage. When the moving and stationary blades have identical shapes and dimensions, the stage is termed a symmetrical stage.

Equations (11), (12), (13), and (16) apply equally well to a reaction stage.

Large turbines are made up of any number of impulse or reaction stages or combinations of impulse and reaction stages.

ASSIGNMENT

1. Determine the work of compression, expansion, heat supplied, and cycle efficiency at 60 F.
2. Determine the pump work, work output, heat supplied, heat rejected, and cycle efficiency for a Rankine cycle operating between 600 psia and 60 F.
3. If the expansion process of problem 2 is 85% efficient, what is the cycle efficiency?
4. Determine the items called for in problem 2 when superheated steam at 600 psia and 740 F is used in place of the saturated steam.
5. Determine the Rankine cycle and Carnot cycle efficiencies for a cycle operating between 300 psig and 15 in. Hg vacuum when the barometer is 29 in. Hg, and steam leaving the boiler has a quality of 96%.
6. Determine the cycle efficiency for a regenerative cycle operating with saturated steam at 150 psia; the condenser pressure is 14.7 psia and the steam is extracted at 50 psia for open feedwater heating.
7. In a reheating cycle, superheated steam at 500 psia and 700 F is expanded to 100 psia, reheated to 700 F, then expanded to 0.95 psia. What is the cycle efficiency?
8. Steam at 362 psia and 600 F is expanded to 14.7 psia, where a portion is extracted for feedwater heating. The remainder of the steam is expanded to 1 psia. What is the cycle efficiency?

George Howey